Review of Right Triangles and Trigonometry using Right Triangles

Proportional Reasoning:

A proportion is created by two equivalents ratios, rates or fractions. You will be required to set up and solve a proportion with correct algebraic steps.

Example:

$$\frac{12}{16} = \frac{x}{100}$$

* Can use cross multiplication to solve

$$\frac{12(100)}{16} = \frac{16x}{16}$$

$$75 = x$$

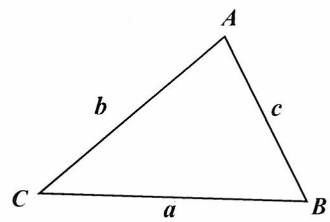
or

* $(100)\frac{12}{16} = \frac{x}{100}(100)$

$$75 = x$$

Triangle Reminders:

- The vertices of a triangle are points so they are labeled using capital letters. The side lengths of a triangle are numbers so they can be labeled using the lower case letter that matches the opposite angle.
- The sum of the angles of a triangle is 180°.
- The largest angle is opposite the longest side. The smallest angle is opposite the smallest side.

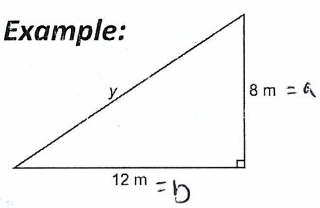


Pythagoras' Theorem:

When working with right triangles, Pythagoras' Theorem can be used to find the length of a side when given the other two sides.

$$a^2 + b^2 = c^2$$

Example:



$$8^2 + 12^2 = y^2$$

$$64 + 144 = y^2$$

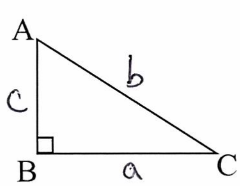
$$\sqrt{208} = \sqrt{y^2}$$

$$14.422\text{m} = y$$

Trigonometric Ratios: $\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

The memory trick we can use to help remember the trigonometric ratios is: SOH CAH TOA

Note: Capital letters represent angles and lower case represent side lengths



sin A = $\frac{a}{b}$ cos C = $\frac{a}{b}$ tan A = $\frac{a}{c}$

To help you decide which trig ratio you are going to use, the following steps will be helpful:

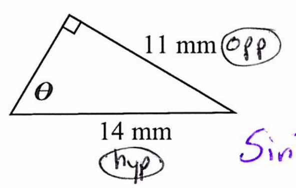
- → Draw and label a diagram if necessary.
- → Identify the hypotenuse.
- → Circle the acute angle you are working with and identify the legs as adjacent or opposite the given angle.
- → Ask yourself "What sides am I using?" then refer back to the trigonometric ratios to decide which to use.
- → Set up the proportion then solve for the missing side or angle.

To find an angle we use the inverse operations $\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$

Examples:

Find the missing value. Round to the nearest tenth of a unit. (Make sure your Calc in on DEG mode)

a)

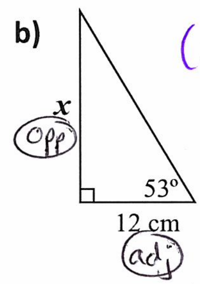


$\sin\theta = \frac{11}{14}$

$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{11}{14}\right)$

$\boxed{\theta = 51.8^\circ}$

b)



$(12)\tan 53^\circ = \frac{x}{12}(12)$

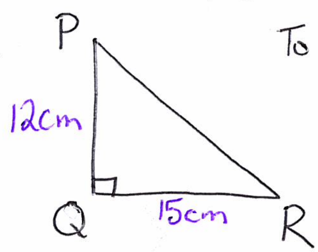
$(12)(\tan 53^\circ) = x$

$\boxed{15.9\text{cm} = x}$

Solving a triangle means to find the measures of the missing sides and angles. You will be given three measures and you will choose the appropriate strategies to determine the remaining three measures.

Example:

Solve ΔPQR, given $\angle Q = 90^\circ$, p = 15 cm and r = 12 cm. Round to the nearest tenth of a unit.



To find q: $q^2 = p^2 + r^2$

$q^2 = 15^2 + 12^2$

$q^2 = 225 + 144$

$\sqrt{q^2} = \sqrt{369}$

$\boxed{q = 19.2\text{cm}}$

To find $\angle R$:

$\tan R = \frac{12}{15}$

$\tan^{-1}(\tan R) = \tan^{-1}\left(\frac{12}{15}\right)$

$\boxed{\angle R = 38.7^\circ}$

Hint: Use the values that are given to solve the unknown values.

To find $\angle P$:

$\tan P = \frac{15}{12}$

$\tan^{-1}(\tan P) = \tan^{-1}\left(\frac{15}{12}\right)$

$\boxed{\angle P = 51.3^\circ}$

Note: Complete answers must include:

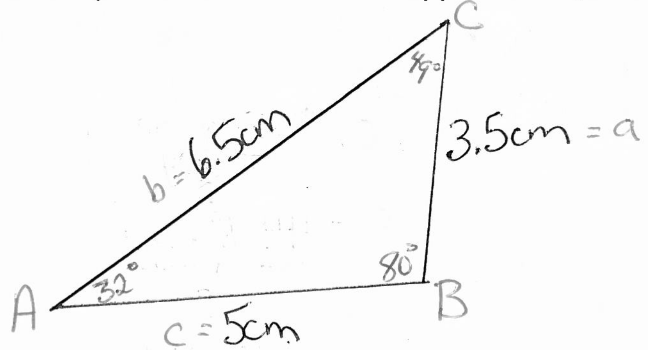
- ✓ Correct notation to identify side or angle
- ✓ Appropriate unit of measure
- ✓ Answer rounded to specified place.
  - o You are required to know words such as tenths, hundredths, etc.!
  - o Using rounded values for further calculations will often lead to incorrect final answers. To avoid this mistake, you must STO answers in your calculator to reach your final answer.

3.2 –The Sine Law for Acute- Oblique Triangles (Concept # 22/24)

REVIEW: We can use the trigonometric ratios (sine, cosine and tangent) and Pythagoras' Theorem to find missing measures in a right triangle. What can we do if we do not have a right triangle?

Define Oblique Triangle- Any triangle that does not contain a right angle.

Activity: Draw an oblique triangle. Label it ΔABC. Measure all sides and angles using a ruler and protractor. (Remember : side "a" is opposite ∠A, side "b" is opposite ∠B, side "c" is opposite ∠C)



Find the ratio of the sine of each angle with its corresponding side: (Round to the nearest thousandth ~~Hundredth~~)

$$\frac{\sin A}{a} = \frac{\sin 32^\circ}{3.5} = 0.1514 \quad \frac{\sin B}{b} = \frac{\sin 80^\circ}{6.5} = 0.1515 \quad \frac{\sin C}{c} = \frac{\sin 49^\circ}{5} = 0.1509$$

The Sine Law: For any triangle, ΔABC where a, b, c are sides opposite ∠A, ∠B, ∠C respectively.

Given ΔABC:

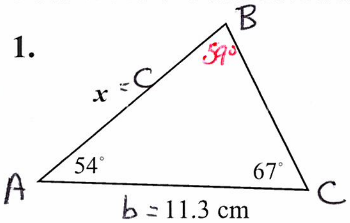
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Note: You use only two of the three ratios to find a missing measure!

Examples:

Determine the indicated side length or angle to the nearest tenth of a unit. **(Concept #22)**

1.

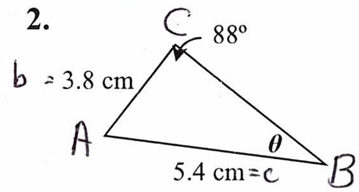


Step 1 = label ΔABC and sides.

Step 2: Find $\angle B = 180-54-67$ ★Need to know one angle and it's opposite side★
$= 59°$

Step 3
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
★recommend if you are finding a side length put as numerator and vice versa for angles★
$$(\sin 67°)\ \frac{11.3}{\sin 59°} = \frac{c}{\sin 67}\ (\sin 67°)$$
$$\boxed{c = 12.1\text{cm}}$$

2.



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$(3.8)\ \frac{\sin\theta}{3.8} = \frac{\sin 88°}{5.4}\ (3.8)$$
$$\sin^{-1}(\sin\theta) = \sin^{-1}\left[\frac{(\sin 88)(3.8)}{5.4}\right]$$
$$\boxed{\theta = 44.7°}$$

In grade 10 Math, you learned that ***solving a triangle*** means to find the measures of the missing sides and angles. You need at least three pieces of information to solve a triangle.

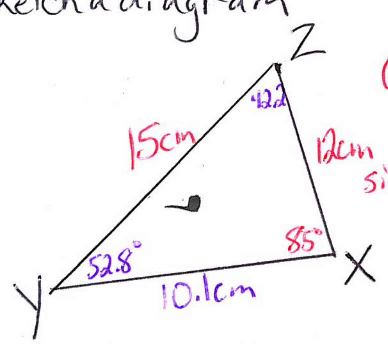
The ***Sine Law*** is used to solve triangles if:

- you know the length of a side of a triangle and the measure of any two angles, you can find the measure of the other two sides.
- you know the length of two sides and the measure of a non-included angle, you can find the measure of the measure of the other non-included angle *provided the triangle can exist**.

Example:

2. Solve ΔXYZ, given $\angle X = 85°$, $x = 15$ cm and $y = 12$ cm. **(Concept #22)**

Sketch a diagram



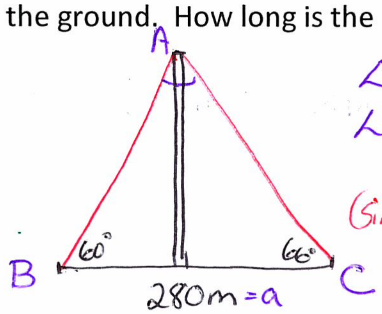
Find $\angle Y$
$$(12)\ \frac{\sin 85}{15} = \frac{\sin Y}{12\text{cm}}\ (12)$$
$$\sin^{-1}\left[\frac{(12)(\sin 85)}{15}\right] = \sin^{-1}(\sin Y)$$
$$\boxed{52.8° = \angle Y}$$

Find $\angle Z$
$$\angle Z = 180 - 85 - \left[\sin^{-1}\left[\frac{12(\sin 85)}{15}\right]\right]$$
$$\boxed{\angle Z = 42.2°} \leftarrow \text{Std } Z$$

Find 'z' side length
$$(\sin Z)\ \frac{15}{\sin 85} = \frac{z}{\sin Z}\ (\sin Z)$$
$$\boxed{10.1\text{cm} = z}$$

3. A radio tower is supported by two wires on opposite sides. On the ground, the ends of the wire are 280 m apart. One wire makes a 60° angle with the ground. The other makes a 66° angle with the ground. How long is the shorter wire? **(Concept #24)**

Draw a diagram



$\angle A = 180 - 60 - 66$

$\angle A = 54°$

$(\sin 66)\dfrac{280}{\sin 54°} = \dfrac{c}{\sin 66}(\sin 66)$

$316.177\text{m} = c$

$(\sin 60)\dfrac{280}{\sin 54} = \dfrac{b}{\cancel{\sin 60}}(\cancel{\sin 60})$

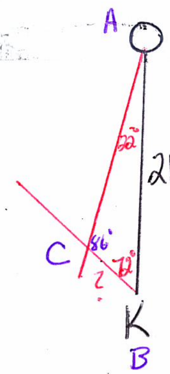
$\boxed{299.731\text{m} = b}$

Note: The smaller angle will be opposite the smaller side.

The shorter wire is 299.731m

Communication | Tip

4 Cape Knox is located 215.0 km due south of Cape Ommaney, British Columbia. A hovercraft leaves Cape Ommaney on a heading of S22°W. A tug boat leaves Cape Knox and travels on a heading of N72°W. How far from Cape Knox will their paths cross?



$\angle C = 180 - 22 - 72$

$\angle C = 86°$

$(\sin 22)\dfrac{215}{\sin 86} = \dfrac{a}{\cancel{\sin 22}}(\cancel{\sin 22})$

$\boxed{80.737\text{Km} = a}$

Their paths will cross 80.737Km from Cape Knox.

Assignment Pg 124 Concept #22 = #2,3af,6ac,7,8 Concept #24 = # 5, 10, 12

4.1/4.2 The Sine law for Obtuse – ObliqueTriangles (Concept #22)

Example 1: Determine the measure of 2 angles between 0-180 that have the following sine ratios. Round to the nearest degree.

a) 0.34 $\sin^{-1}(0.34)$

$\sin 160 = 0.342$ ✓ $\theta_1 = 20°$

$\theta_2 = 180 - 20 = 160°$

Two angles are 20° and 160°

b) 4/7 $\sin^{-1}\left(\frac{4}{7}\right)$

$\boxed{\theta_1 = 35°}$ $\theta_2 = 180 - 35$ $\boxed{\theta_2 = 145°}$

check

$\sin 35 = 0.5735$

$\sin 145 = 0.5735$

✓ $\frac{4}{7} = 0.5735$

Ex. 2) Determine the measure of angle C to the nearest degree.

$$(11)\ \frac{\sin 48°}{9} = \frac{\sin C}{11}\ (11)$$

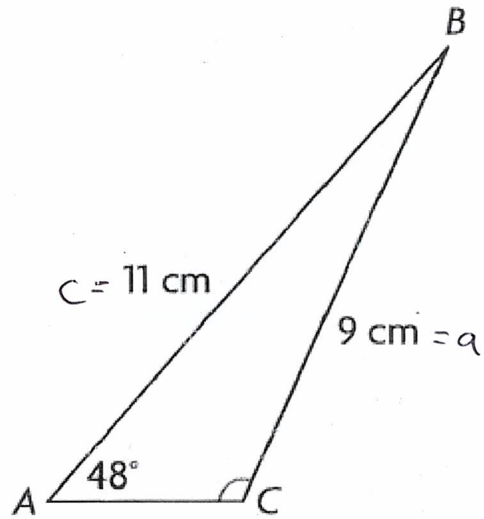
$$\sin^{-1}\left[\frac{(11)(\sin 48)}{9}\right] = \sin^{-1}(\sin C)$$

$$65.2698\ldots = \angle C$$

Sto C

$180 - C$ because angle c appears to be obtuse.

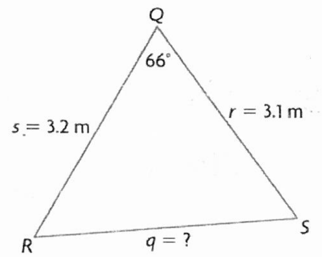
$$\boxed{115° = \angle C}$$

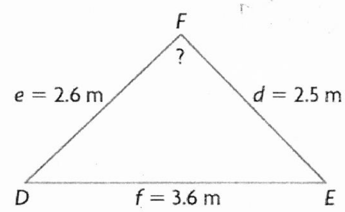


Assignment Pg 173 #15, pg 175 #8 Pg 163 #3

3.3/4.2 – The Cosine Law for Oblique Triangles (Concept #22/24)

Use the Sine Law to write the relationship of the three pairs of sides and opposite angles for each triangle, then solve for the unknown values.





Is there a problem? Need to know at least one angle and the side length opposite it.

The Sine Law will work only with certain types of given information:

- If you know the length of a side of a triangle and the measure of any two angles, you can find the measure of the other two sides. (ASA or AAS triangles)
- If you know the length of two sides and the measure of an angle opposite a known side, you can find the measure of the measure of the angle opposite the other known side *provided the triangle can exist**. (SSA triangles)

What type of information is provided in the two given triangles? What is the missing measure?

- (SAS) Two sides and the angle between them are given. Missing two ∠'s and one side
- (SSS) Given all side lengths missing all angles.

We will now use Pythagoras' Theorem to derive a new relationship that will work with the two given triangles.

$h^2 = b^2 - x^2 \qquad h^2 = c^2 - y^2$

$b^2 - x^2 = c^2 - y^2$ $\quad +x^2 \quad +x^2$

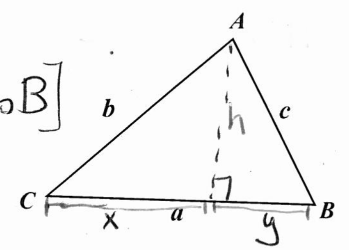
$b^2 = c^2 - y^2 + x^2$

$b^2 = c^2 - y^2 + (a-y)^2$

$b^2 = c^2 - y^2 + a^2 - ay - ay + y^2$

$b^2 = c^2 + a^2 - 2ay$

$\triangleright\ b^2 = c^2 + a^2 + 2a[c \cos B]$



$x = (a-y)$

$(c) \cos B = \frac{y}{c} (c)$

$c(\cos B) = y$

The Cosine Law:

Given ΔABC:

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

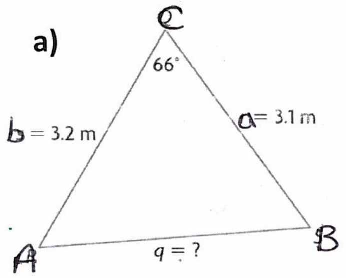
$c^2 = b^2 + a^2 - 2ba \cos C$

**Note:
- $\angle C$ does not always have a measure of 90º!
- The largest angle is always opposite the largest side.

Examples:

1. Determine the indicated side length or angle to the nearest tenth of a unit

a)



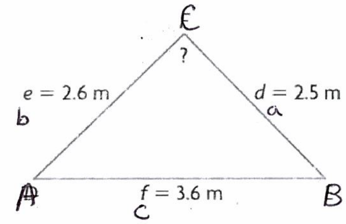
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 3.1^2 + 3.2^2 - 2(3.1)(3.2)\cos 66°$

$\sqrt{c^2} = \sqrt{19.85 - 19.84(\cos 66°)}$

$\boxed{c = 3.4\text{m}}$ $\therefore\ q = 3.4\text{m}$

b)



$c^2 = a^2 + b^2 - 2ab\cos C$

$3.6^2 = 2.5^2 + 2.6^2 - 2(2.5)(2.6)\cos C$

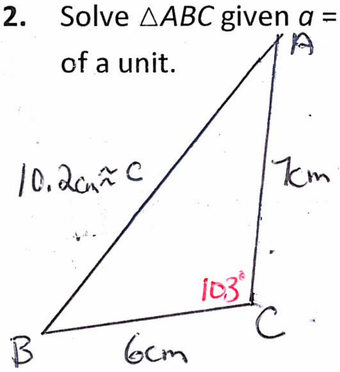
$12.96 - 13.01 = 13.01 - 13.01 - 13(\cos C)$

$\dfrac{-0.05}{-13} = \dfrac{-13(\cos C)}{-13}$

$\cos^{-1}\left[\dfrac{0.05}{13}\right] = \cos^{-1}(\cos C)$

$\boxed{\angle F = 89.8°}$

2. Solve $\triangle ABC$ given a = 6 cm, b = 7 cm and $\angle C$ = 103°. Round all answers to the nearest tenth of a unit.



$c^2 = 7^2 + 6^2 - 2(7)(6)\cos 103°$

$\sqrt{c^2} = \sqrt{85 - 84(\cos 103°)}$

$\boxed{c = 10.2\text{cm}}$

$(7)\dfrac{\sin 103}{10.2} = \dfrac{\sin B}{7}(7)$

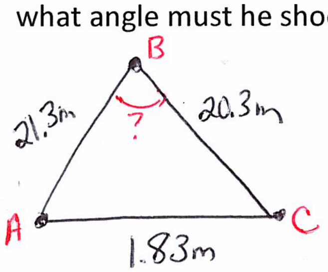
$\sin^{-1}\left[\dfrac{7(\sin 103)}{10.2}\right] = \sin^{-1}[\sin B]$

$\boxed{42.0° = \angle B}$ $\boxed{\text{Sto B}}$

$\angle A = 180 - 103 - B$

$\boxed{\angle A = 35.0°}$

3. During a hockey game, a player on the blue line shoots a puck toward the 1.83-m-wide net from a point that is 20.3 m from one goal post and 21.3 m from the other goal post. Within what angle must he shoot to hit the net? Answer to the nearest tenth of a degree.



$1.83^2 = 20.3^2 + 21.3^2 - 2(20.3)(21.3)\cos B$

$3.3489 - 865.78 = 865.78 - 865.78 - 864.78\cos B$

$\dfrac{3.3489 - 865.78}{-864.78} = \dfrac{-864.78\cos B}{-864.78}$

$\cos^{-1}\left[\dfrac{3.3489 - 865.78}{-864.78}\right] = \cos^{-1}(\cos B)$

$\boxed{\angle B = 4.2°}$

Assignment **Pg 138 #2,3,4a,5b,7bc (Concept #22)** **Pg 138 # 8,10,11 (CONCEPT #24)**