– Sine Law : The Ambiguous Case

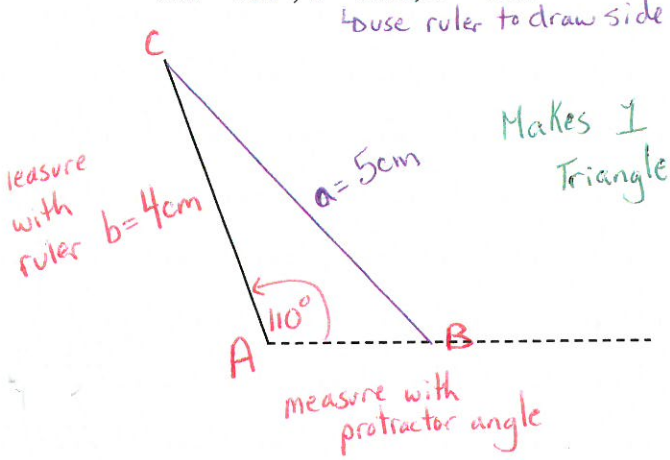
The Sine Law gives the relationship between the sides and the angles of a triangle. If you are given two angles and one side, you can use the Law of Sines to find the lengths of the two unknown sides (ASA or AAS triangles).

You can get into trouble when you are given two sides and one opposite angle (SSA triangles) in order to find the other opposite angle as the combination of side lengths and the angle measure does not always produce one unique triangle. Due to this uncertainty, this is called the Ambiguous Case.

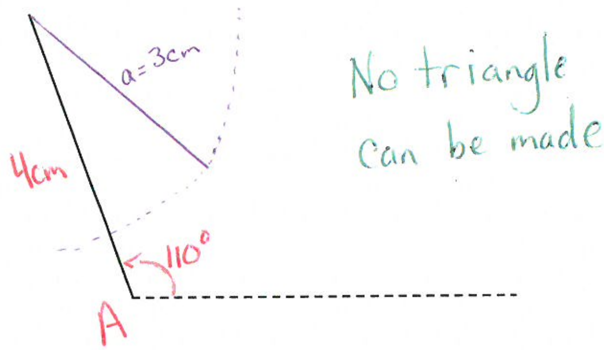
Construct triangle ΔABC given the following information. ☆Use Compass and ruler☆

$\angle A = 110°$, $a = 5\,\text{cm}$, $b = 4\,\text{cm}$

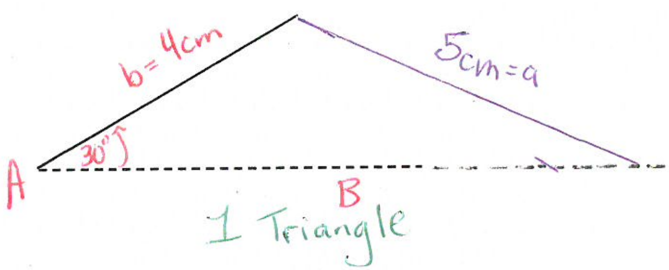
↳use ruler to draw side "a"



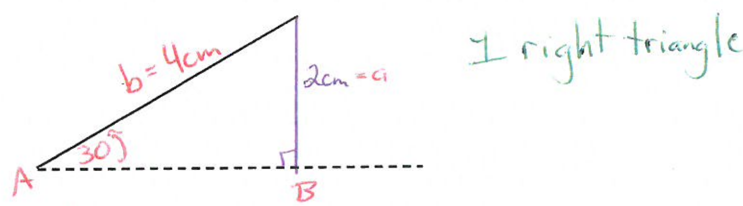
$\angle A = 110°$, $a = 3\,\text{cm}$, $b = 4\,\text{cm}$



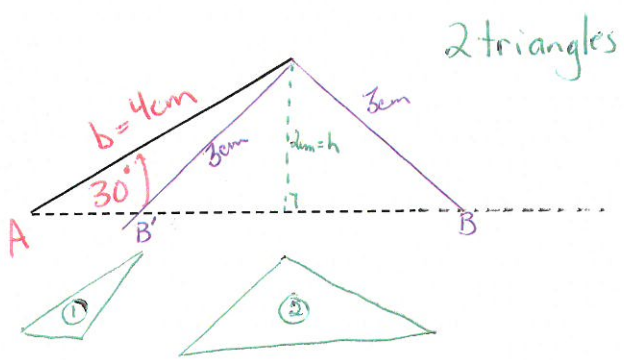
$\angle A = 30°$, $a = 5\,\text{cm}$, $b = 4\,\text{cm}$



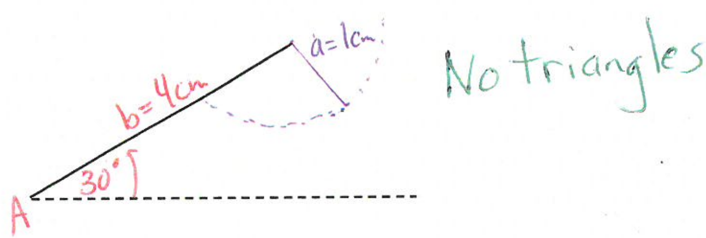
$\angle A = 30°$, $a = 2\,\text{cm}$, $b = 4\,\text{cm}$



$\angle A = 30°$, $a = 3\,\text{cm}$, $b = 4\,\text{cm}$



$\angle A = 30°$, $a = 1\,\text{cm}$, $b = 4\,\text{cm}$



Three different situations can arise when given SSA or ASS information: one triangle formed, two triangles formed or no possible triangles. In order to determine the number of triangles, you must first consider the type of angle then the lengths of the adjacent and opposite sides.

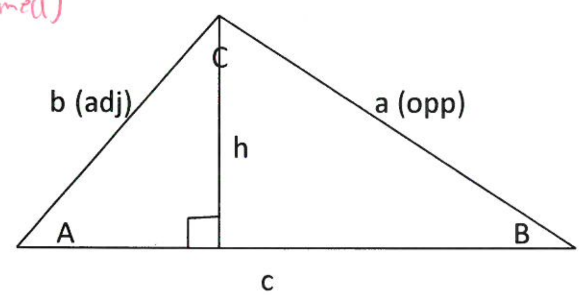
Type of Angle
↓
Relationship of the Given Sides
↓
Number of Triangles that can be formed
↓
Number of Solutions

General Conclusions

ACUTE TRIANGLES:

Given sides "a" and "b" and ∠A (where "a" is the side opposite the known ∠)

If $a \geq b$ (opp≥adj) then 1 solution (1 triangle formed)



If $a < b$(opp < adj), we have the following possibilities

- $a < h$ (opp< (b)(sinA)) No triangles
- $a = h$ (a = (b)(sinA)) 1 right triangle
- $a > h$ (a>(b)(sinA)) 2 triangles

(where b sinA = altitude to $\overline{AB}$ from *C*)

$\sin A = \dfrac{h}{b}$

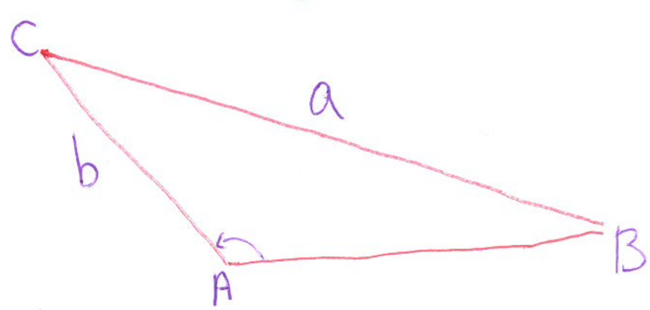
h = b sinA

OBTUSE TRIANGLES:

Given 2 Sides and an ***Obtuse*** Non-Included Angle

When ∠ A is <u>obtuse</u>, we have the following additional cases:

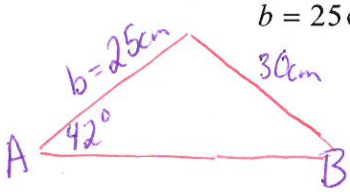
- If $a > b$ (opp > adj), there is 1 solution. (1 triangle)
- If $a \leq b$ (opp ≤ adj), there is 0 solution. (No triangle formed)



Examples: **(Concept #23)**

1. Determine the number of solutions for ΔABC given following SSA information. Sketch the triangles. :

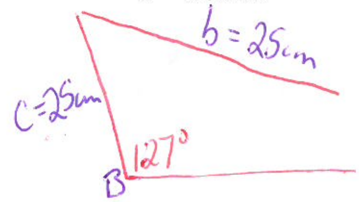
a) $\angle A = 42°$, $a = 30\,\text{cm}$ and $b = 25\,\text{cm}$



opp > adj

$\boxed{\text{1 solution}}$

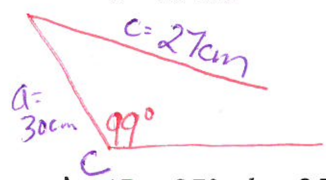
d) $\angle B = 127°$, $b = 25\,\text{cm}$ and $c = 25\,\text{cm}$



$\boxed{\text{0 solutions}}$

Obtuse angle and opp = adj

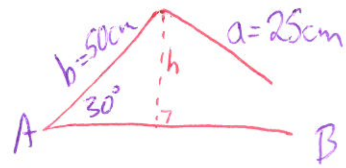
b) $\angle C = 99°$, $a = 30\,\text{cm}$ and $c = 27\,\text{cm}$



$\boxed{\text{0 solutions}}$

obtuse angle and opp < adj

e) $\angle A = 30°$, $a = 25\,\text{cm}$ and $b = 50\,\text{cm}$



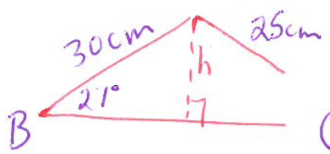
opp < adj check height

$(50)\sin 30 = \frac{h}{50} \cdot 50$

$h = 25$

$\boxed{\text{1 Solution}}$

c) $\angle B = 27°$, $b = 25\,\text{cm}$ and $c = 30\,\text{cm}$



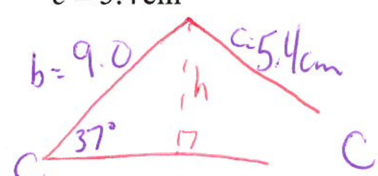
opp < adj check height

$(30)\sin 27 = h$

$h = 13.61$

$\boxed{\text{2 Solutions}}$

f) $\angle C = 37°$, $b = 9.0\,\text{cm}$ and $c = 5.4\,\text{cm}$



opp < adj check heigh

$9(\sin 37) = h$

$h = 5.41$

$\boxed{\text{1 right } \Delta}$

2. Solve ΔBAD given $\angle B = 37.7°$, $b = 30\,\text{cm}$ and $d = 42\,\text{cm}$. Round all angle measures and side lengths to the nearest hundredth.

Assign. Pg 183 #1-5