7.3,4,5,7,8: Applications of Quadratics Functions in Standard Form $y = ax^2+bx+c$ – Concept #10

Example 1: A ball is thrown into the air and follows the path given by $h(t) = -5t^2 + 20t + 1$, where *h* represents height in meters and *t* represents time in seconds.

a) Determine ***the initial height*** of the ball. The initial height will occur when time is 0 secs.

$h(0) = -5(0)^2 + 20(0) + 1$

$h(0) = 1$ meter The initial height is 1m, which is also the y-int. (h-intercept in this case) of the graph.

b) Determine the ***vertex***.

Step 1: Find t-intercepts (x-int.) by factoring or quadratic formula

Step 2: Find the equation of the axis of symmetry, which is the x-value (t-value) of the vertex

Step 3: Substitute the x-value (t-value) into the function to find the y-value (h-value) of the vertex.

$\boxed{\text{Step 1}}$ $0 = -5t^2 + 20t + 1$ ← Not easily factorable so use the quadratic formula

$t = \frac{-20 \pm \sqrt{20^2 - 4(-5)(1)}}{2(-5)}$

$t = \frac{-20 \pm \sqrt{400 + 20}}{-10}$

$t = \frac{-20 \pm \sqrt{420}}{-10}$

$t = \frac{-20 + \sqrt{420}}{-10}$ ← Sto in graphing calc "A"

$t = \frac{-20 - \sqrt{420}}{-10}$ ← Sto in graphing calc "B"

$\boxed{\text{Step 2:}}$ $t = \frac{A + B}{2}$

$t = 2$ ← equation of axis of symmetry and t-value of vertex

$\boxed{\text{Step 3:}}$ h-coordinate:

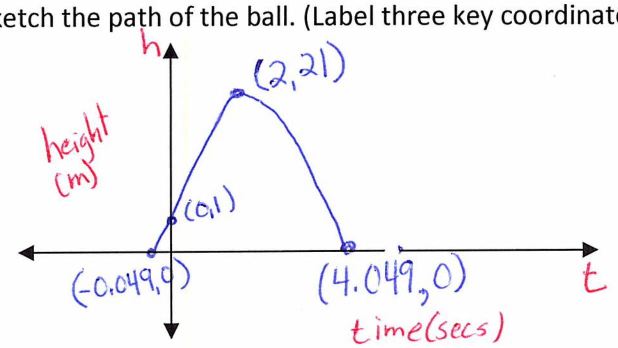
$h(2) = -5(2)^2 + 20(2) + 1$

$h(2) = -20 + 40 + 1$

$h(2) = 21$

$\boxed{\text{vertex} = (2, 21)}$

c) Sketch the path of the ball. (Label three key coordinates and the axes.)



d) What is the ball's ***maximum height***? 21m

e) How long does it take for the ball to reach its ***maximum height***? 4.049 secs $\approx \frac{-20 - \sqrt{420}}{-10}$

f) What is the height of the ball after 3 seconds?

$h(3) = -5(3)^2 + 20(3) + 1$

$h(3) = -45 + 60 + 1$

$h(3) = 16$m

g) What are the domain and range of this function?

$D = \{t \mid 0 \le t \le 4.049, t \in \mathbb{R}\}$

$R = \{h \mid 0 \le h \le 21, h \in \mathbb{R}\}$

Example 2 (Pg 407 #14)

Samuel is hiking along the top of the First Canyon on the South Nahanni River in the Northwest Territories. When he knocks a rock over the edge, it falls into the river, 1260m below. The height of the rock, h(t), at t seconds can be modelled by the following function: $h(t) = -25t^2 - 5t + 1260$

a) How long will it take the rock to reach the water? Note: height of rock equals 0m when it reaches the water

$0 = -25t^2 - 5t + 1260$

$0 = -5(5t^2 + 1 + 252)$ ← solve by factoring

$0 = -5(5t + 36)(t - 7)$

$5t + 36 = 0$ $t - 7 = 0$

$5t + 36 - 36 = 0 - 36$ $t - 7 + 7 = 0 + 7$

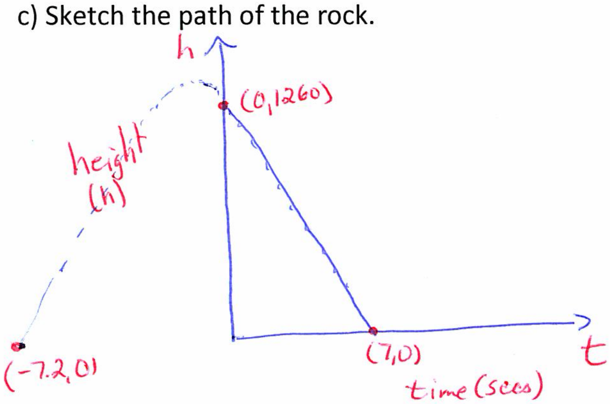
$\frac{5t}{5} = \frac{-36}{5}$ $t = 7$

$t = \frac{-36}{5} = -7.2$ $t = 7$

↑ Inadmissible solution as time is not negative

The rock will take 7 seconds to reach the water

c) Sketch the path of the rock.



d) What is the domain and range of the function?

$D = \{t \mid 0 \le t \le 7, t \in \mathbb{R}\}$

$R = \{h \mid 0 \le h \le 1260, h \in \mathbb{R}\}$

d) Demonstrate the solution using your graphing calculator

Example 3- Pg 374 Ex.1

The flight time for a long-distance water ski jumper depends on the initial velocity of the jump and the angle of the ramp. For one particular jump, the ramp has a vertical height of 5 m above water level. The height of the ski jumper in flight, $h(t)$, in metres, over time, t, in seconds, can be modelled by the following function:

$$h(t) = 5.0 + 24.46t - 4.9t^2$$

a) How long does this water ski jumper hold his flight pose?

The skier holds his flight pose until he is 4.0 m above the water.

b) What is the highest height the ski jumper reaches? Use technology to help you answer these question

a) $4 - 4 = 5 - 4 + 24.46t - 4.9t^2$

$0 = -4.9t^2 + 24.46t + 1$ ← graph on graphing calculator and find zeros.

$t = 5.0323$ seconds

b) on graphing calculator calculate the maximum value

$h = 31.525$ m

Assignment: Pg 371 #12-14 Pg 407 #13(Solve by factoring) Pg 428 #8, 10 Pg 380 #7,9 (Using Graphing Calc)