## 7.6 Application of Quadratics using Vertex Form $y = a(x-h)^2 + k$ or $y = a(x-p)^2 + q$

**Example 1:** Given the equation $y = x^2 + 4$. If the graph is shifted down 2 units, which equation describes the new graph?

the y-coordinate of the vertex will go down 2.

a) $y = x^2 + 6$ b) $y = (x-2)^2 + 2$ c) $y = (x-2)^2 + 4$ d) $y = x^2 + 2$

Vertex $(0,4) \downarrow 2$ $(0,2)$ ← new vertex

**Example 2:** Given $y = -2(x-3)^2$ If the function is shifted 8 units to the right and 3 units up, write an equation that describes the new function.

x-coordinate will increase by 8 y-coordinate will increase by 3

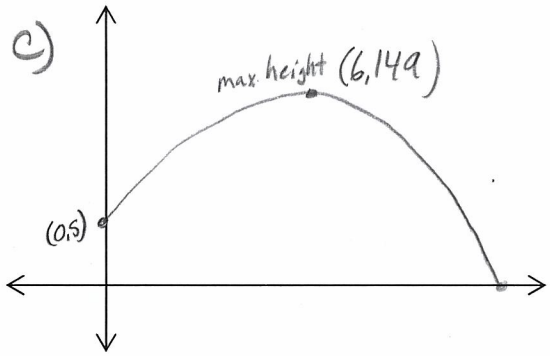
$y = -2(x-3)^2 + 0$ (+8, +3)

$y = -2(x-11)^2 + 3$

**EXAMPLE 3:** A toy rocket is shot up in the air from a hill. Its height in meters above ground, *h*, is recorded after *t* seconds. The path the rocket follows is given by the following equation: $h(t) = -4(t-6)^2 + 149$

a) Write the function in $y = ax^2 + bx + c$ form.
b) What is the initial height of the rocket?
c) Sketch the path of the rocket. (Label your sketch)
d) When will the rocket reach its maximum height?
e) What is the maximum height of the rocket?
f) How long does the toy rocket remain in the air for?

c)




a) $h(t) = -4(t-6)^2 + 149$

$h(t) = -4(t-6)(t-6) + 149$

$h(t) = (-4t+24)(t-6) + 149$

$h(t) = -4t^2 + 24t + 24t - 144 + 149$

$h(t) = -4t^2 + 48t + 5$

b) Initial height is the y-intercept 5m

d) Vertex (6, 149)
The rocket will reach its maximum height at 6secs

e) The maximum height the rocket reaches is 149m

f) Need to calculate x-intercepts.

$x = \frac{-48 \pm \sqrt{(48)^2 - 4(-4)(5)}}{2(-4)}$

$x = \frac{-48 \pm \sqrt{2304 + 80}}{-8}$

$x = \frac{-48 \pm \sqrt{2384}}{-8}$

$x = -0.103$ $x = 12.103$

The rocket is in the air for 12.103secs.

**EXAMPLE 4:**

A cable that hangs between two telephone poles makes a parabola shape that has the equation $y = \frac{1}{10}(x-9)^2 - 7$

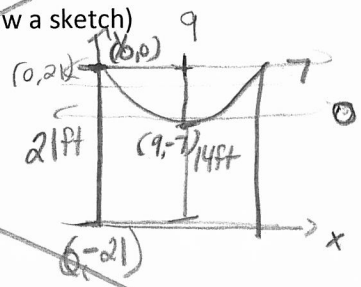
where x and y are measured in feet. If the cable is attached to both poles at a height of 21 feet and the lowest point of the cable is 14 feet above the ground, how far away are the poles from each another? ( Draw a sketch)

$+7 = \frac{1}{10}(x-9)^2$

$\sqrt{70} = (x-9)^2$

$\pm\sqrt{70}+9 = x = 17.36$

18

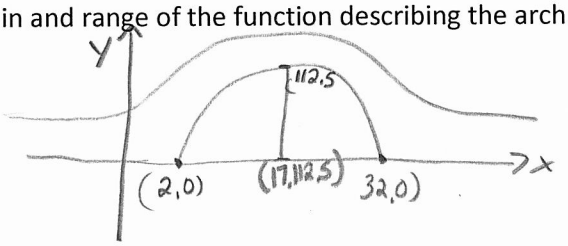



**Example 5:**

A bridge with a parabolic archway has zeros located at (2, 0) and (32, 0).The arch has a maximum height of 112.5 ft.

a) Determine the equation of the archway in vertex form.

b) State the domain and range of the function describing the arch.




$y = a(x-h)^2 + k$ $y = ax^2 + bx + c$

$y = a(x-17)^2 + 112.5$

$0 = a(2-17)^2 + 112.5$

$0 = a(225) + 112.5 - 112.5$

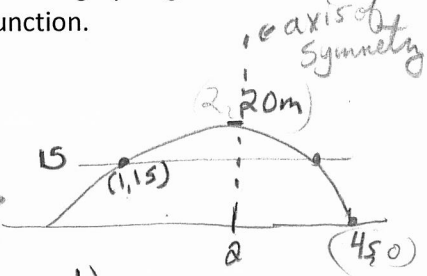
$\frac{-112.5}{225} = \frac{225a}{225}$

$-0.5 = a$

$y = -0.5(x-17)^2 + 112.5$

**Example6:** A soccer ball is kicked from the ground. After 2 seconds, the ball reaches its maximum height of 20 m. It lands on the ground at 4s.

a) Determine the quadratic function that models the height of the kick. Write it in vertex- graphing form.
b) Determine any restrictions that must be placed on the domain and range of the function.
c) What is the height of the ball after 1 s?
d) When was the ball at the same height on the way down?




a) $y = a(x-k)^2+q$
$y = a(x-2)^2+20$
$0 = a(4-2)^2+20$
$0 = a(4)+20-20$
$\frac{-20}{4} = \frac{4a}{4}$
$-5 = a$
$y = -5(x-2)^2+20$

b) $\{x \mid 0 \le x \le 4, x \in \mathbb{R}\}$
$\{y \mid 0 \le y \le 20, y \in \mathbb{R}\}$

c) $y = -5(1-2)^2+20$
$y = -5(-1)^2+20$
$y = -5(1)+20$
$y = 15$
The ball is 15m above the ground at 1 second.

d) at 3 secs

**Assignment**

1. Given $y - 1 = 2(x+1)^2$. If the equation is shifted left 5 units, which equation describes the new graph? (vertex $(-1,0)$; x-coordinate $-5$)
   a. $y - 6 = 2(x-4)^2$ c. $y - 1 = 2(x+6)^2$ e. $y - 1 = 2(x-6)^2$
   b. $y - 1 = 2(x-4)^2$ d. $y - 1 = 2(x+5)^2$
2. If the given function $y = (x-1)^2 + 3$ is shifted up 3 units and left 4 units, which equation describes the new graph? (vertex $= (1,3)$; $+3$ to $y$; $-4$ to $x$; New vertex $(-3,6)$)
   a. $y = (x-4)^2 + 3$ c. $y - 7 = (x+2)^2$ e. $y = (x+3)^2 + 6$
   b. $y - 3 = (x+4)^2$ d. $y = (x-5)^2$
3. If the given function $y = (x+3)^2 + 4$ is shifted down 5 units, which equation describes the new function? (vertex $(-3,4)$; y value $-5$)
   a. $y = (x+3)^2 + 9$ c. $y = (x+8)^2 + 4$ e. $y = -5(x+3)^2 + 4$
   b. $y = (x+3)^2 - 1$ d. $y = (x-2)^2 + 4$
4. Given $y = x^2$. If the function is shifted 4 units to the left, write an equation that describes the new function. (x-coordinate $-4$)

$y = (x+4)^2$

**Pg 420 #13, 15, 16, 17 – 19 ( Choose 2 questions from 17-19 )**