

### 7.5 Solving Quadratic Equations by Factoring

To solve an equation like  $ax^2 + bx + c = y$  means to find the values of the variable that causes the left side of the equation to equal the right side. If  $y=0$ , then we're really just solving for the x-intercepts.

Remember: Zeros/ roots are just asking for the x intercepts.

**Example 1** Solve the by factoring  $2x^2 - 14x - 60 = 0$  and verify the solutions

GCF = 2

Factor  $2(x^2 - 7x - 30) = 0$

$2(x + 3)(x - 10) = 0$

$x + 3 = 0$        $x - 10 = 0$

$x = -3$        $x = 10$

zero product property: Set each factor equal to zero

To verify your solutions substitute answers into the original equation to see if it satisfies the equation.

$x = \{-3, 10\}$  ← solution set

**Example 2** Determine the roots of the following equations.

a)  $75p^2 - 192 = 0$

$3(25p^2 - 64) = 0$

$3(5p - 8)(5p + 8) = 0$

$5p - 8 = 0$        $5p + 8 = 0$

$\frac{5p}{5} = \frac{8}{5}$        $\frac{5p}{5} = \frac{-8}{5}$

$p = \frac{8}{5}$        $p = \frac{-8}{5}$

$x = \{\frac{8}{5}, \frac{-8}{5}\}$  or  $\{1.6, -1.6\}$

c)  $0 = -2(2x - 7)(4x - 1)$

$2x - 7 = 0$        $4x - 1 = 0$

$\frac{2x}{2} = \frac{7}{2}$        $\frac{4x}{4} = \frac{1}{4}$

$x = \frac{7}{2}$        $x = \frac{1}{4}$

$x = \{\frac{7}{2}, \frac{1}{4}\}$  or  $\{3.5, 0.25\}$

b)  $4x^2 + 28x + 49 = 0$

$2x \begin{array}{l} \nearrow 7 \\ \searrow 7 \end{array} \left| \begin{array}{l} 14x \\ 14x \\ \hline 28x \end{array} \right.$

Perfect Square trinomial  
∴ only one solution and one x-intercept

$(2x + 7)(2x + 7) = 0$

$2x + 7 = 0$

$\frac{2x}{2} = \frac{-7}{2}$

$x = \frac{-7}{2}$

$x = \{-\frac{7}{2}\}$  or  $\{-3.5\}$

d)  $1.4y^2 + 5.6y - 16.8 = 0$

$\frac{(10)}{10} \frac{14}{10} y^2 + \frac{(10)}{10} \frac{56}{10} y - \frac{(10)}{10} \frac{168}{10} = 0$

$14y^2 + 56y - 168 = 0$

$7(2y^2 + 8y - 24) = 0$

$7 \cdot 2(y^2 + 4y - 12) = 0$

$14(y + 6)(y - 2) = 0$

$y + 6 = 0$        $y - 2 = 0$

$y = -6$        $y = 2$

$x = \{-6, 2\}$

**Example 3** a) Mrs. Hock claimed that she solved a quadratic equation by graphing. She found the zeros were

$-\frac{4}{3}$  and 5. Find **AN** equation that she might have solved. (Remember that many parabolas have the same x-intercepts.)

What variable distinguishes between these different parabolas?

Pick any "a" value

$$Y = a(3x+4)(x-5)$$

$$Y = 2(3x+4)(x-5)$$

$$(3) X = -\frac{4(3)}{3} \quad X = 5 - 5$$

$$3x = -4 + 4 \quad X - 5 = 0$$

$$3x + 4 = 0$$

b) Find THE equation of the parabola that has the same x-intercepts as question (a) but passes through the point

(2,3)  
x y

$$y = a(3x+4)(x-5)$$

$$3 = a(3(2)+4)(2-5)$$

$$3 = a(10)(-3)$$

$$\frac{3}{-30} = \frac{-30a}{-30}$$

$$-\frac{1}{10} = a$$

$$Y = -\frac{1}{10}(3x+4)(x-5)$$

**Example 4** Suppose your best friend solved the quadratic equation as shown:

$$4x^2 = 9x$$

$$\frac{4x^2}{x} = \frac{9x}{x}$$

$$4x = 9$$

$$x = \frac{9}{4} \text{ or } 2.25$$

← set equation equal to zero first.

Divided both sides by x

← can't divide by \* because x could = 0.

Is this solution correct? If not, identify the error. Then, solve the quadratic correctly.

No,

Correct solution

$$4x^2 = 9x - 9x$$

$$4x^2 - 9x = 0$$

$$x(4x - 9) = 0$$

$$x = 0 \quad 4x - 9 = 0 + 9$$

$$\frac{4x}{4} = \frac{9}{4}$$

$$x = 2.25$$

$$X = \{0, 2.25\}$$