7.4 Factored Form of a Quadratic Function Concept #7/8

Zeros: The value(s) which make an expression equal to zero

Roots: The value(S) that are the solution(s) to a mathematical equation

X- Intercepts: Points on the graph of a relation where the relation crosses the x-axis. These are the points for which the y-value is 0.

The x-intercepts are also called the **'zeros'** of the function, or the **'roots'** of the function.

Zero Product Property: If $a \times b = 0$ then a = 0 or b= 0 (ex./ (x-n)(x-m) =0 then x-n=0 or x-m=0)

Example 1 Sketch the graph of the quadratic function $f(x) = 2x^2 + 14x + 12$ by first (Concept #7)

a) Completely factoring the right side of the equation (Remember to look for a GCF first).

$f(x) = 2(x^2 + 7x + 6)$

$f(x) = 2(x+6)(x+1)$

b) Find the zeros of the function. (Let y=0 & solve for x using the zero product property)

$\frac{0}{2} = \frac{2(x+6)(x+1)}{2}$

$0 = (x+6)(x+1)$

$x+6 = 0$ $x+1 = 0$

$x = -6$ $x = -1$

b) Find the y-intercept (let x = 0 and solve for y)

yint = 12

(0,12)

c) Find the vertex of the function & identify the equation of the axis of symmetry.

If x-intercepts are (-6,0) and (-1,0) find the average

$x = \frac{-6 + -1}{2}$

$x = -3.5$ (equation of axis of symmetry)

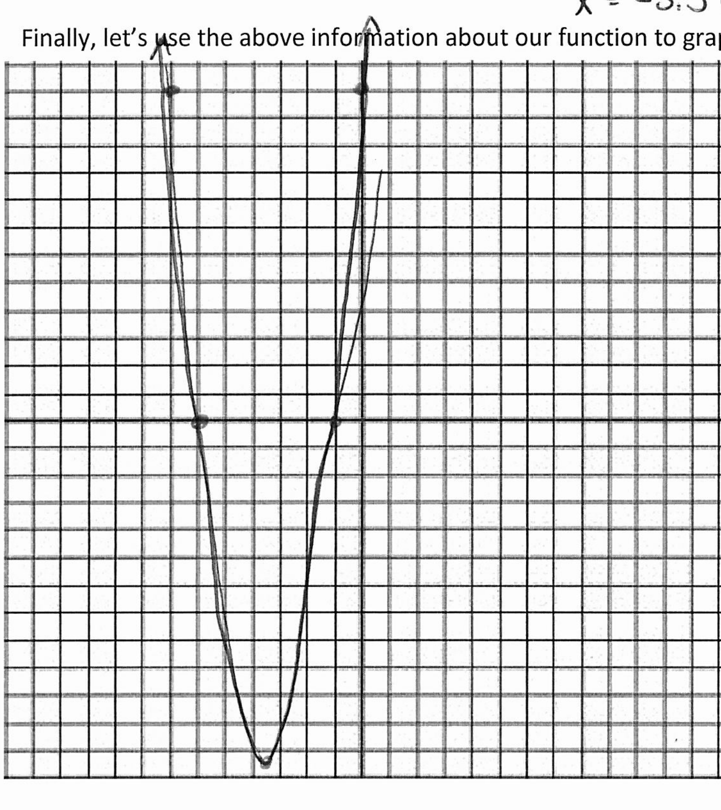
Find y if x = -3.5 to find vertex

$y = 2(-3.5)^2 + 14(-3.5) + 12$

$y = 24.5 + (-49) + 12$

$y = -12.5$

c) Finally, let's use the above information about our function to graph it...



Vertex: $(-3.5, -12.5)$

Y – Intercept: (0,12)

X – Intercept: (-6,0) (-1,0)

Equation of Axis of Symmetry: $x = -3.5$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq -12.5, y \in \mathbb{R}\}$

Notice that when the zeros were at x = -1 and x = -6 that the equation of the parabola in factored form looked like:

$$y = 2(x+1)(x+6)$$

A quadratic function is in factored form when it is written in the form y =a(x-r)(x-s), where r and s represent the values of the zeros.

Since r = -1 and s = -6, $y = 2(x-r)(x-s)$ becomes

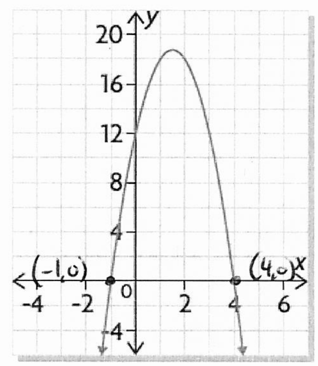
$$y = 2(x-(-1))(x-(-6))$$

$$y = 2(x+1)(x+6)$$

<u>Example 2</u> - Determine the function that defines the parabola. Write the function in standard form $y = ax^2 + bx + c$ (Concept #8)

Step 1: Find the x-intercepts

$(4,0) \quad (-1,0)$



Step 2: Write the factored form of the quadratic function,

substituting the zeros for r and s

$$y = a(x-r)(x-s)$$

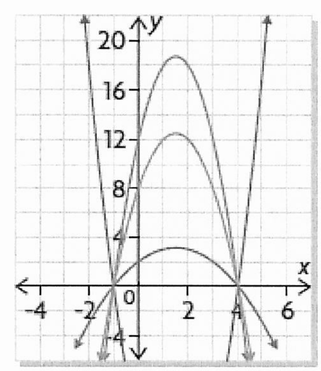
$$y = a(x-4)(x-(-1))$$

$$y = a(x-4)(x+1)$$

Step 3: Since there are infinitely many parabolas that have these two zeros (ours is the tallest parabola facing downward), we need to find the value of a.

- Choose any other point that you know **lies on** the parabola.
  This point will serve as an (x,y) point that satisfies the quadratic equation.

  Since the y-intercept is 12, we'll use the point (0,12)



$$y = a(x-4)(x+1)$$

$$12 = a(0-4)(0+1)$$

$$\frac{12}{-4} = \frac{a(-4)(1)}{-4}$$

$$-3 = a$$

In factored form, the quadratic function is:

$$y = -3(x-4)(x+1)$$

In standard form, the quadratic function is:

$$y = (-3x+12)(x+1)$$

$$y = -3x^2 - 3x + 12x + 12$$

$$y = -3x^2 + 9x + 12$$

Pg 392 #6 together using Graphing Calculators **Assignment: p. 391 #1,3, 4acdf, 5acdf, 11**