6.4 Creating the model for Optimization Problems (Concept #4)

KEY NEW IDEAS FOR THIS LESSON

You will be able to develop algebraic and graphical reasoning by solving optimization problems using linear programming

OPTIMIZATION PROBLEM: A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

CONSTRAINT: A limiting condition of the optimization problem being modelled, represented by a linear inequity.

OBJECTIVE FUNCTION: In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized

FEASIBLE REGION: The solution region for a system of linear inequalities that is modelling an optimization problem.

OPTIMAL SOLUTION: A point in the solution set that represents the maximum or minimum value of the objective function. If a vertex isn't included in the feasible region the optimal solution will be a point, within the feasible region that is close to the vertex

LINEAR PROGRAMMING: A mathematical technique used to determine which solutions in the feasible region result in the optimal solutions of the objective function.

Steps to Solving an Optimization Problem:

- The solution to an optimization problem is usually found at one of the vertices of the feasible region.
- To determine the optimal solution to an optimization problem using linear programming, follow these steps:

Step 1. Create an algebraic model that includes:
- a defining statement of the variables used in your model
- the restrictions on the variables
- a system of linear inequalities that describes the constraints
- an objective function that shows how the variables are related to the quantity to be optimized

Step 2. Graph the system of inequalities to determine the coordinates of the vertices of its feasible region. → Just going to step 2 in 6.4

Step 3. Evaluate the objective function by substituting the values of the coordinates of each vertex.

Step 4. Compare the results and choose the desired solution.

Step 5. Verify that the solution(s) satisfies the constraints of the problem situation.

Note: In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities.

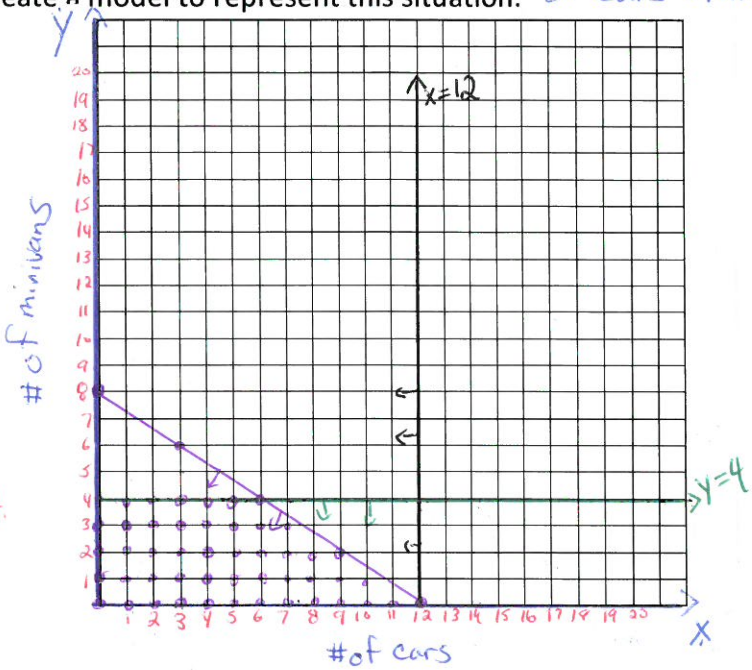
EXAMPLE #1:

Three teams are travelling to a basketball tournament in cars and minivans.

* Each team has no more than 2 coaches and 14 athletes = 16 team members x 3 = 48 people
* Each car can take 4 team members, and each minivan can take 6 team members.
* No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

Possible Combinations of Cars + Minivans



1. Variable Statement:

Let x = # of cars
y = # of minivans

2. Domain, Range and Restrictions:

$x \in W$ $y \in W$

3. Constraint Inequalities:

$4x + 6y \le 48$ purple → $\frac{6y}{6} \le \frac{-4x}{6} + \frac{48}{6}$

$y \le -\frac{2}{3}x + 8$

slope y-int.

$x \le 12$ Black

$y \le 4$ green

4. Graph the above Constraint Inequalities

within the restrictions:

5. Objective Function to be Maximized/Minimized:

Let V = total # of vehicles $x + y = V$

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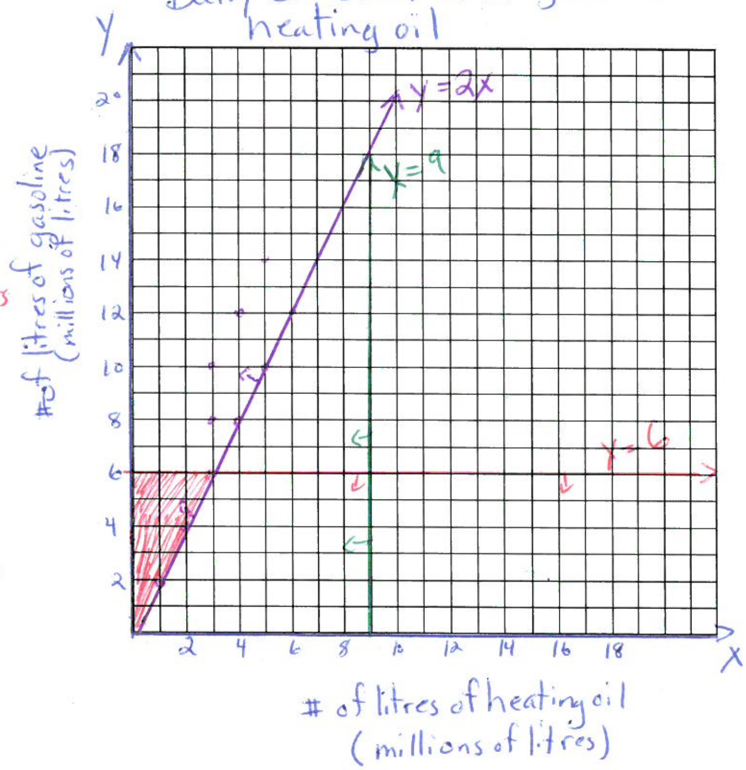
EXAMPLE #2:

A refinery produces oil and gas.

* At least 2 L of gasoline is produced for each litre of heating oil
* The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
* Gasoline is projected to sell for $1.10 per litre.
* Heating oil is projected to sell for $1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

Daily combination of gas and heating oil



1. Variable Statement:

Let x = # of litres of heating oil
y = # of litres of gasoline

2. Domain and range and restrictions:

$x \in \mathbb{R}$ $y \in \mathbb{R}$, $x \leq 0$, $y \leq 0$ ← these restrictions become a part of your constraints

3. Constraint Inequalities:

$x \leq 0$ $y \leq 0$

$2x \leq y$

$x \leq 9\,000\,000$

$y \leq 6\,000\,000$

4. Graph the above Constraint Inequalities within the restrictions:

5. Objective Function to be Maximized/Minimized:

Let R = Total ~~rev~~ daily revenue

$1.75x + 1.10x = R$

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ASSIGNMENT:

Note: Please do each question on one side of a page. Please make sure that there are at least 10 lines that are blank on the bottom of each page as we will be doing a bit more work on these questions tomorrow!

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