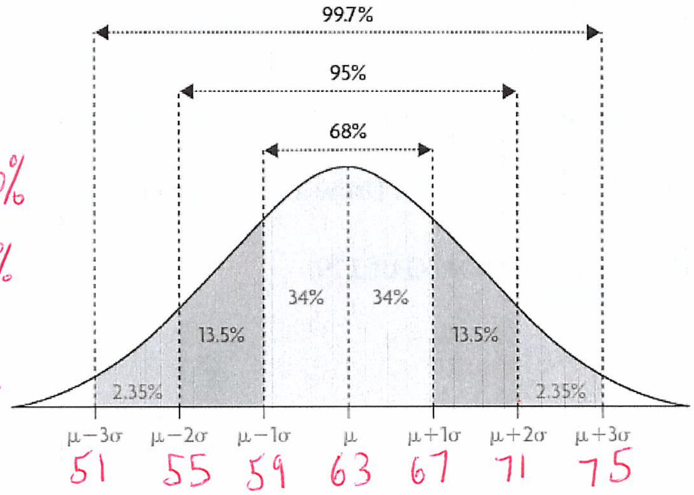
5.5 Z- Scores (Concept #27 and 28)

Page 251 #1:

"Normal Curve" or "Bell Curve" – Normal Distribution



Mean = 63 years

S.D. = 4 years

What % of curlers are 63 years or younger? 50%

What % of curlers are older than 71 years? 2.5%

What % of curlers are 61 years or younger?

Need to calculate z-scores.

What can we do when we need an area in between deviations?!

Z-scores is a standardized value that indicates the number of standard deviations of a data value above or below the mean. When standardizing you are converting the distributions to a **standard normal distribution** which has a mean of zero and a standard deviation of one.

Z score: $$z = \frac{x - \mu}{\sigma}$$

Note: Round z score to two decimal places

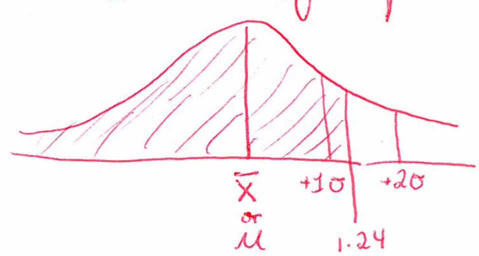
Example 1:
Determine the z-score for the value of x: $\mu = 165$, $\sigma = 48$, $x = 36$

$$z = \frac{36-165}{48} = -2.6875$$

Example 2:
Determine the percent of the data to the left of a z-score of 1.24

Use Chart to find percentages

0.8925×100
$= 89.25\%$



Example 3:
Determine the percent of the data to the right of a z-score of -2.35

look up -2.35 on the z-score chart.

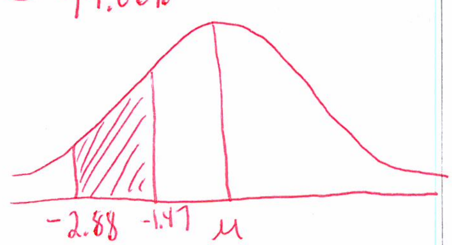
0.0094 ← % to the left
$\times 100 = 0.94\%$

$100 - 0.94$
$= 99.06\%$

or $1 - 0.0094$
$= 0.9906 \times 100$
$= 99.06\%$

Example 3:
Determine the percent of data between z-scores of -2.88 and -1.47

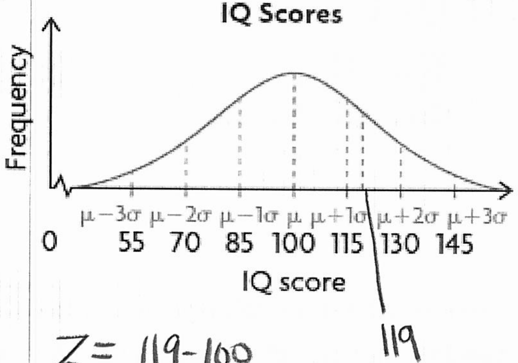
$(\% \text{ of } -1.47) - (\% \text{ of } -2.88)$
$= 0.0708 - 0.0020$
$= 0.0688 \times 100 = 6.88\%$

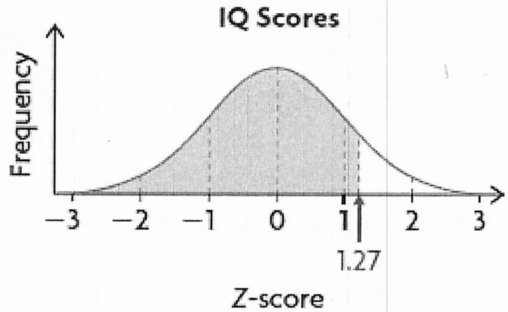


Example 4 IQ tests are sometimes used to measure a person's intellectual capacity at a particular time. IQ scores are normally distributed, with a mean of 100 and a standard deviation of 15. If a person scores 119 on an IQ test, how does this score compare with the scores of the general population?

Step 1: Sketch a normal curve and determine the IQ scores for one, two and three deviations from the mean. Draw a line to identify where 119 falls on the curve.

Step 2: Determine the z-score for an IQ of 119.





$$Z = \frac{119-100}{15}$$

$$Z = 1.27$$

Step 3: Determine the area under curve (the percentage of people with an IQ **<u>less than</u>** 119). To do this we use a **z-score table**. Look on **<u>page 592, 593 in the textbook.</u>**

z	0.0	0.01	0.06	0.07
0.0	0.5000	0.5040	0.5239	0.5279
0.1	0.5398	0.5438	0.5636	0.5675
1.1	0.8643	0.8665	0.8770	0.8790
1.2	~~0.8849~~	~~0.8869~~	~~0.8962~~	**0.8980**
1.3	0.9032	0.9049	0.9131	0.9147

z-score table
A table that displays the fraction of data with a z-score that is less than any given data value in a standard normal distribution.

The value in the z-score table is 0.8980. This means that an IQ score of 119 is greater than 89.8% of IQ scores in the general population.

Example 5

Athletes should replace their running shoes before the shoes lose their ability to absorb shock.

Running shoes lose their shock-absorption after a mean distance of 640 km, with a standard deviation of 160 km. Zack is an elite runner and wants to replace his shoes at a distance when only 25% of people would replace their shoes. At what distance should he replace his shoes?

$\bar{X} = 640\text{Km}$

$\sigma = 160\text{Km}$

$Z \rightarrow 25\%$

Look up z-score backwards

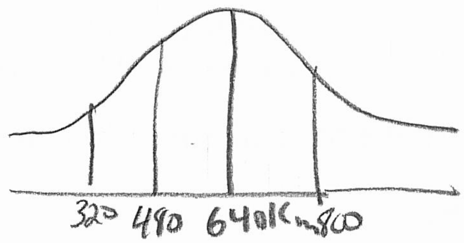
$Z = -0.67 \rightarrow$ when 25% of people would replace their shoes

$$Z = \frac{X - \bar{X}}{\sigma}$$

$$(160)\,-0.67 = \frac{X - 640}{160}\,(160)$$

$$\overset{+640}{-107.2} = X - 640 + 640$$

$$\boxed{532.8 = X}$$



Zack should replace his running shoes after 533 km.

Day 1 Practice: p. 264 # 1,2,6,9a,10,14

Example 6

The ABC Company produces bungee cords. When the manufacturing process is running well, the lengths of the bungee cords produced are normally distributed, with a mean of 45.2 cm and a standard deviation of 1.3 cm. Bungee cords that are shorter than 42.0 cm or longer than 48.0 cm are rejected by the quality control workers.

a) If 20 000 bungee cords are manufactured each day, how many bungee cords would you expect the quality control workers to reject?

Step 1: Determine the z-scores for the minimum and maximum acceptable lengths.

Minimum length = 42 cm

$$z_{min} = \frac{x - \mu}{\sigma}$$

$$= \frac{42 - 45.2}{1.3}$$

$$z_{min} = -2.461$$

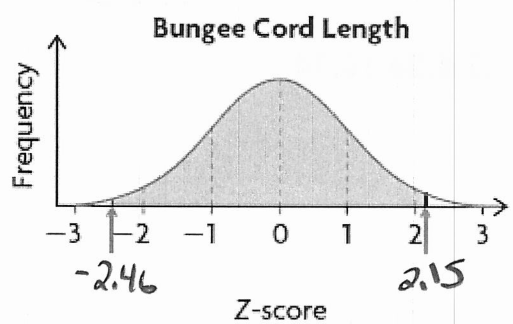
Maximum length = 48 cm

$$z_{max} = \frac{x - \mu}{\sigma}$$

$$= \frac{48 - 45.2}{1.3}$$

$$z_{max} = 2.153$$

Step 2: Sketch the normal curve.



The area under the curve to the left of -2.46 represents the percent of rejected bungee cords that are less than 42 cm. The area under the curve to the right of 2.15 represents the percent of rejected bungee cords that are greater than 48 cm.

Step 3: Find the area under the curves using the z-score table.

Area to the left of -2.46 = 0.0069

Area to the right of 2.15 = 1 – area under the curve that is less than 2.15

1 - 0.9842
= 0.0158

Percent rejected = area to the left of 0.0069 + area to the right of 0.0158

= 0.0069 + 0.0158
= 0.0227 = 2.27%

Total number of bungee cords rejected =

0.0227 x 20000
= 454 bungee cords

Example 7 Determining warranty periods

A manufacturer of personal music players has determined that the mean life of the players is 32.4 months, with a standard deviation of 6.3 months. What length of warranty should be offered if the manufacturer wants to restrict repairs to less than 1.5% of all the players sold?

Step 1: Using the z-score table, find the score that closely relates to an area under the curve of 1.5% (0.015).

Z = -2.17

Step 2: Substitute the known values into the z-score formula and solve for x.

$$Z = \frac{x-\mu}{\sigma}$$

$$(6.3)\,-2.17 = \frac{x-32.4}{6.3}\,(6.3)$$

$$-13.671 + 32.4 = x - 32.4 + 32.4$$

$$18.729 = x$$

The manufacturer should offer an 18 month warranty. (round down because he wants to repair less than 1.5%)

Day 2 Practice: p. 264 # 3, 4, 7, 8a, 9b, 11, 16, 20