Foundations 20 Final Exam Review - Answer Key

1. TRUE OR FALSE.

a) F a conjecture is a testable expression that is NOT proven to be true.
b) T
c) F a counter example INVALIDATES a conjecture.
d) T
e) T
f) T
g) F The more support you have for a conjecture, the stronger it is.
h) F You must prove your conjecture true for all cases
i) F
j) T
k) T
l) F Circular reasoning is NOT a reliable way to prove your conjecture

2. odd + even = sum

collect evidence
$3+4=7$
$5+6=11$
$21+22=43$
$-3+6=3$

make a conjecture:
The sum of one odd integer and one even integer will result in an odd integer

test your conjecture.
$15+18=33$
$101+100=201$

3. $(\text{odd})^2 = \text{odd}$
$(3)^2 = 9$
$(-5)^2 = 25$
$(11)^2 = 121$
$(1)^2 = 1$
Yes, Paula's conjecture is reasonable.

4.

choose #	6	10
$\times 2$	12	20
$+6$	18	26
$\times 2$	36	52
-4	32	48
$\div 4$	8	12
-2	6	10

conjecture:
The # you end up with was the # you started with.

5. n
$2n$
$2n+6$
$4n+12$
$4n+8$
$n+2$
n

6. (odd integer)^2 - (the odd integer) = even.

inductive examples:

a) $(3)^2 - (3) =$
$9 - 3 = 6$

b) $(5)^2 - 5$
$25 - 5 = 20$

deductive PROOF: Let $2x+1$ = any odd integ

$(2x+1)^2 - (2x+1)$
$4x^2 + 4x + 1 - (2x+1)$
$4x^2 + 4x + 1 - 2x - 1$
$4x^2 + 2x =$
$2(2x^2 + x)$ ✓ divisible by two means its even.

7. If Adrian's pants are not Khaki they would be a different kind of pants then we cannot assume they are expense

8.

2	7	6
9	5	1
4	3	8

2	9	4
7	5	3
6	1	8

9. Andy Bonnie Candice Darlene

Scenarios: A C B / B C A } based on clue #1

based on clue #3 → * D A C B / B C A D } based on clue #2

Darlene, Andy, Candice, Bonnie.

10. a) 180°
b) equal
c) transversal
d) Interior
e) outside
f) corresponding
g) alternate interior, =
h) Supplementry

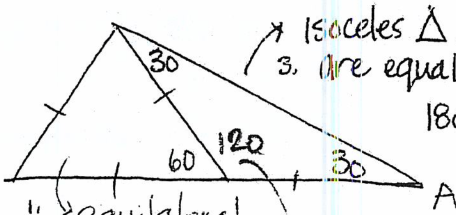
11. In order to prove that two lines are //
1) Corresponding angles must be =.
2) alternate interior angles must be =.
3) alternate exterior angles must be =.
4) Same side interior angles must be supplementry

12. Vertically opposite angles being = DOES NOT prove that two lines are //.

13. a) Corresponding angles are =, so lines are //
b) Same side interior angles are NOT supplementry, so line are NOT //.
c) Alternate exterior angles are =, so lines are //.
d) same as c)

14. a) $\angle a = 75°$ b/c corresponding $\angle$
$\angle c = 105°$ b/c supplementry to 75°/definition of a line.
$\angle d = 105°$ b/c vertically opposite to c + supplementry to 75°
$\angle b = 105°$ b/c corresponding to c + alternate exterior to d.

15.



3. Isoceles Δ so base angles are equal.

$180 - 120 = 60$ $\frac{60}{2} = 30$

$\angle A = 30°$

1. equilateral So all angles = 60°

2. $180 - 60 = 120$ def'n of a line

14.b) $\angle b = 55°$, $\angle c = 75°$
$\angle a = 50°$, $\angle d = 75°$
$\angle e = 55°$, $\angle f = 50°$

14.c) $\angle y = 60°$, $\angle x = 50°$

14.d) $\angle a = 129°$, $\angle b = 51°$

16. $\angle DAN = 53°$ because it is supplementary to 127°
$\angle NDA = 29°$ the sum of the interior angles in a Δ is 180°
$\angle SYD = \angle NDA = 29°$
$\overline{SY} \parallel \overline{AD}$ Corresponding angles are equal

17. lines are $\parallel$ so,
a) $\angle c = 76°$ (vertically opposite)
$\angle b = \angle c =$ alternate exterior
$\angle a = 180 - 76 = 104°$

b) $2a + 3a = 180$ (same side int)
$\frac{5a}{5} = \frac{180}{5}$
$a = 36°$

$b = 3a$
$b = 3(36)$
$b = 108°$

$c = b$ (alt exterior)
$c = 108°$

18. $\angle y = 180 - 40 - 45$ } interior angles of a Δ
$y = 95°$

$\angle x = 40°$ } line $\parallel$ so alt int $\angle$'s are =
$\angle z = 45°$

19. $180(n-2) =$
$180(20-2) =$
$180(18) = 3240°$

20. $\frac{180(n-2)}{180} = \frac{3060°}{180}$
$n - 2 = 17$
$+2 \quad +2$
$n = 19$ sides

21.a) $\frac{180(n-2)}{n} = 140°$

$180(n-2) = 140n$

$180n - 360 = 140n$

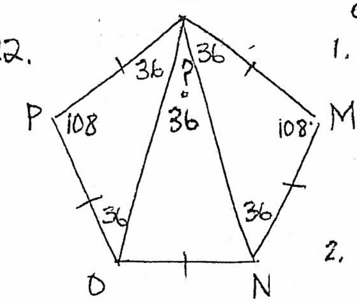
$\frac{-360}{-40} = \frac{-40n}{-40}$

$9 = n$

b) if the interior angle is 140°, the one exterior angle is 40°

$9 \times 40° = 360°$

22.



each interior ∠.

1. $\frac{180(5-2)}{5} = 108°$

2. $180 - 108 = 72$

$\frac{72}{2} = 36°$

3. $108 - 36 - 36 = 36°$

23. exterior ∠'s of a convex polygon have a sum of 360°.

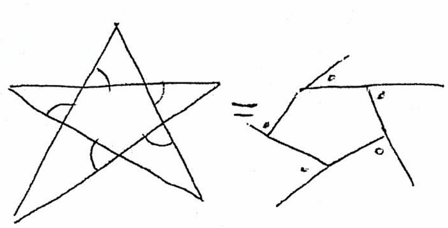
$70 + 60 + 90 + x + x = 360$

$2x = 140$

$x = 70°$

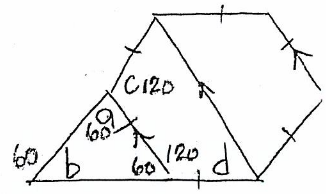
The interior angles would then be 110, 120, 90°, 110°, 110°

24.



exterior angles add to 360°

25. a)



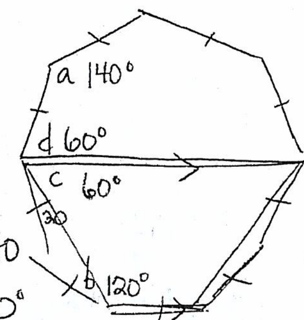
$\angle c = \frac{180(6-2)}{6} = 120°$

$\angle a = 180 - 120° = 60°$

$\angle b = 60°$

$\angle d = 60$ corresponding to 60°

b)



$\frac{180(9-2)}{9}$

140°

$\angle a = 140°$

$180 - 140 = 40$

$\frac{40}{2} = 20°$

$\angle b = 20°$

$\angle c = 180 - 120$

$\angle c = 60°$

$\angle d = 140 - 20 - 60$

$\angle d = 60°$

26.a) $2x + 50 + x + 35 + 2x = 180$

$5x + 85 = 180$

$\frac{5x}{5} = \frac{95}{5}$ $\quad x = 19$

b) $130 = 3x + 2x$

$130 = 5x$

$26 = x$

exterior angle is equal to the sum of the two interior non-adjacent angles

#27)

Statement	Reason
① $AB \parallel ED$	① Given
② $\overline{BC} \cong \overline{DC}$	② Given
③ $\angle ACB \cong \angle ECD$	③ Vertically opposite angles
④ $\angle ABC \cong \angle EDC$	④ Alternate interior ang
⑤ $\triangle ABC \cong \triangle EDC$	⑤ ASA
⑥ $\overline{AB} \cong \overline{ED}$	⑥ CPCTC

28. $\sin 50\,(80) = \sin 60\,(w)$

a) $\dfrac{61.28}{.8660} = \dfrac{.8660\,(w)}{.8660}$

$7.1 = w$

b) $\sin 72\,(6) = \sin M\,(10)$

$\dfrac{5.7}{10} = \dfrac{\sin M(10)}{10}$

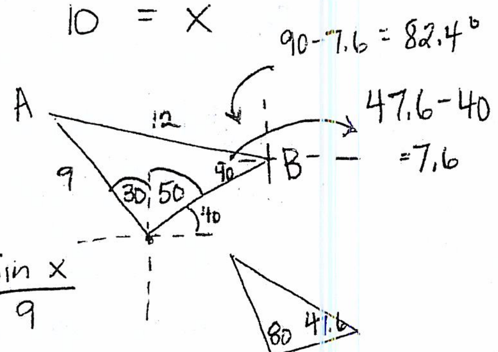
$.5706 = \sin M$

$34.8^\circ = M$

29. $\dfrac{\sin 80^\circ}{12} = \dfrac{\sin 55^\circ}{x}$

$\sin 55\,(12) = \sin 80\,(x)$

$10 = x$

30. 

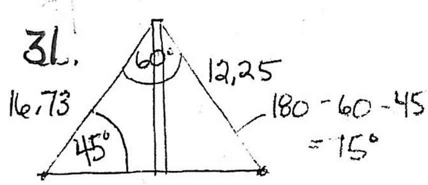
$\dfrac{\sin 80}{12} = \dfrac{\sin x}{9}$

$8.863 = 12 \sin x$

$.7386 = \sin x$

$47.6^\circ = x$

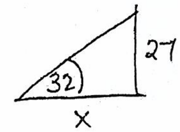
The captain should head N82°W from light house B

31. 

$\dfrac{\sin 60}{15} = \dfrac{\sin 45}{x}$ $\qquad$ $\dfrac{\sin 60}{15} = \dfrac{\sin 75}{x}$

$x = 12.25$ $\qquad$ $x = 16.73$

height = $\sin 45 = \dfrac{h}{16.73} = 11.83$

32. a) 

$\tan 32 = \dfrac{27}{x}$

$.6249 = \dfrac{27}{x}$

$\dfrac{27}{.6249} = x$

$43.2 = x$

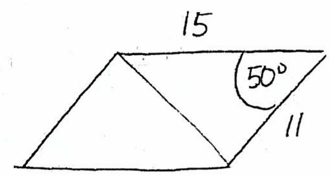
b) $\tan 43 = \dfrac{h}{43.2}$

$.9325\,(43.2) = h$

$40.3 = h$

height of crane = $40.3 - 27$

$= 13.3$ m

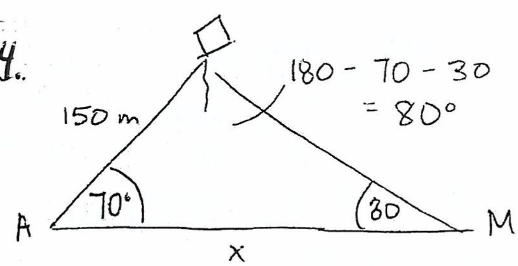
33. 

$a^2 = 15^2 + 11^2 - 2(15)(11)(\cos 50)$

$a^2 = 225 + 121 - 212.$

$a^2 = 134$

$a = 11.6$

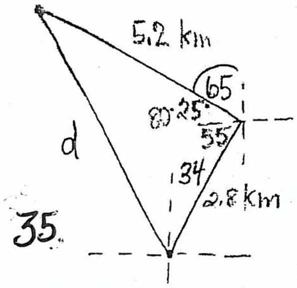
34. 

$\dfrac{\sin 30}{150} = \dfrac{\sin 80}{x}$

$\dfrac{147.7}{.5} = \dfrac{.5x}{.5}$

$295.4 = x$

35.



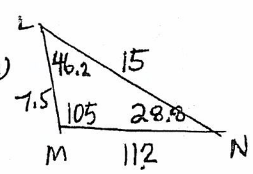
$$d^2 = (5.2)^2 + (2.8)^2 - 2(5.2)(2.8)\cos 80$$
$$d^2 = 29.8$$
$$d = 5.5 \text{ Km} = \text{distance}$$

$$\frac{\sin 80}{5.5} = \frac{\sin x}{5.2}$$
$$\frac{5.121}{5.5} = \frac{5.5(\sin x)}{5.5}$$
$$.9311 = \sin x$$
$$68.6° = x$$

$$68.6 - 34 = 34.6°$$
N 35 W = direction

36. a) .64 ⟶ 40° and 140°
b) $\frac{1}{3}$ ⟶ 19.5° and 160.5°
c) .95 ⟶ 71.8° and 108.2°
d) $\frac{7}{23}$ ⟶ 17.7° and 162.3°

37. a)

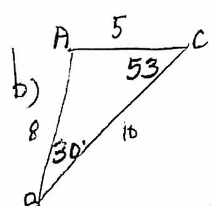


$$\frac{\sin 105}{15} = \frac{\sin L}{11.2}$$
$\angle L = 46.2°$

$$x^2 = 7.5^2 + 11.2^2 - 2(7.5)(11.2)(\cos 105)$$
$$x^2 = 225.2$$
$$x = 15$$

$$\frac{\sin 105}{15} = \frac{\sin N}{7.5}$$
$28.8 = \angle N$

b)

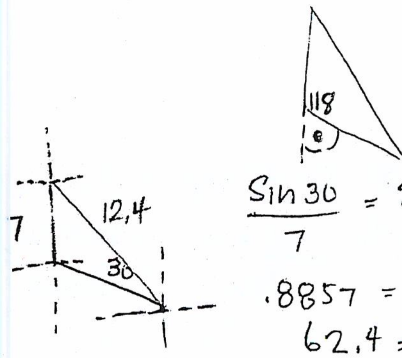


$$\frac{\sin 30}{5} = \frac{\sin C}{8}$$
$53° = C$

$$\cos B = \frac{5^2 - 10^2 - 8^2}{-2(10)(8)}$$
$\cos B = .86875 = 30°$

$\angle A = 180 - 30 - 53$
$= 97°$

38.



$$\frac{\sin 30}{7} = \frac{\sin \angle}{12.4}$$
$.8857 = \angle$
$62.4 = \angle$ or $180 - 62.4$ $117.6°$

• $180 - 118 = 62°$

Compass direction S 62°E will get them back to the parking lot.

39. a) 2 △'s, or 2 solutions
b) 1 △, or 1 solution
c) Zero △'s
d) 1 △
e) 1 right △

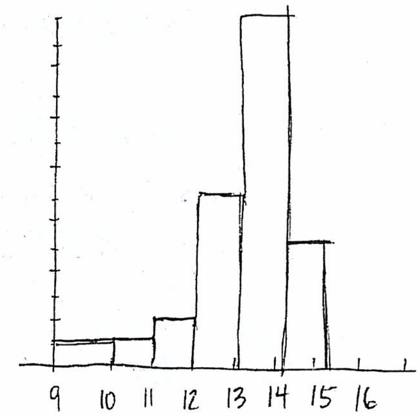
9.6, 10.4, 11.5, 11.8, 12.8, 12.8, 12.8, 12.9, 12.9, 12.9, 13, 13.1, 13.1, 13.2, 13.2, 13.3, 13.3, 13.4, 13.5, 13.5, 13.6, 13.6, 13.6, 13.8, 13.9, 14.1, 14.4, 14.5, 14.6, 14.8

40. mean = 13.13
median = 13.25
mode = 12.8, 12.9 + 13.6

range = 14.8 − 9.6 = 5.2
σ = 1.1007

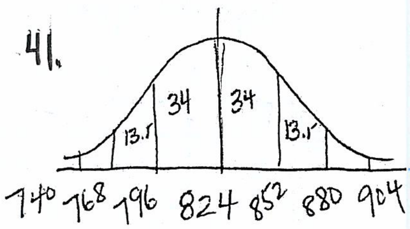
d) The range & σ are low which tells us that the data is not spread out too far and is relatively consistent.

interval	tally	freq
9.0 - 10.0	I	1
10 - 11.0	I	1
11.0 - 12.0	II	2
12 - 13.0	𝍸 II	7
13 - 14.0	𝍸 𝍸 IIII	14
14 - 15.0	𝍸	5



c) the data is somewhat normally distributed.

41.

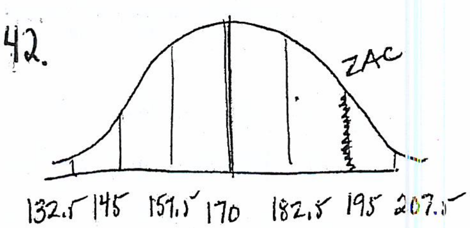


b) 34 34 = 68%
796 824 852

c) 2.35 13.5 34 = 15.85%
740 768 796 824

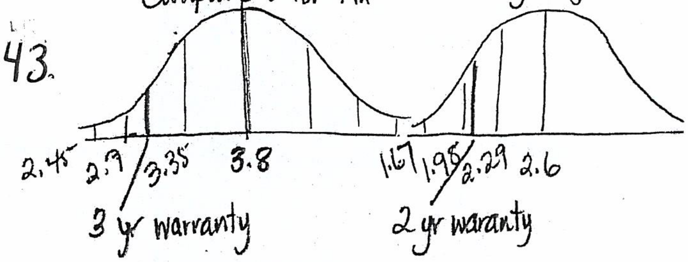
d) 95% = between 768 km & 880 km

42.



a) 50 + 34 + 13.5 = 97.5% shorter
b) 100 − 97.5 = 2.5% taller.

43.



$Z = \frac{3 - 3.8}{.45}$ $\qquad$ $Z = \frac{2 - 2.6}{.31}$

$Z = -1.78$ $\qquad$ $Z = -1.94$

= use your table =

.0375 $\qquad$ .0262
3.75% $\qquad$ 2.62%

Computers For All is more likely to fail before its warranty is over because there is more area to the left.

44. Female: $Z = \frac{277 - 247}{33} = .9091$

Male: $Z = \frac{499 - 461}{51} = .7451$

The female had the greatest mass when compared to the other bears b/c its weight is further from the mean

45. a) margin of error $= \pm 3.1\%$

b) confidence level $= \frac{19}{20}$ or 95%

c) i) health care $= 23.1 \pm 3.1\%$

$20\% - 26.2\%$

ii) environment $= 12.6 \pm 3.1\%$

$9.5\% - 15.7\%$

46. dashed — Solution region

~~45.~~ a)

$x + y > 4$

$y > -x + 4$

dashed

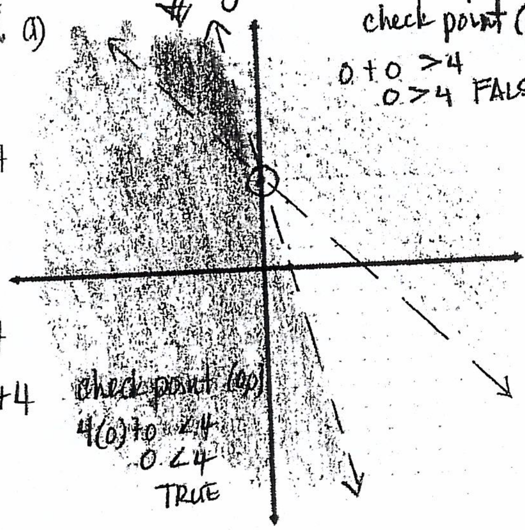
$4x + y < 4$

$y < -4x + 4$

check point (0,0)

$0 + 0 > 4$

$0 > 4$ FALSE



b)

$x + y \leq 1$

$y \leq -x + 1$

Solid Line

$0 + 0 < 1$

$0 < 1$ TRUE

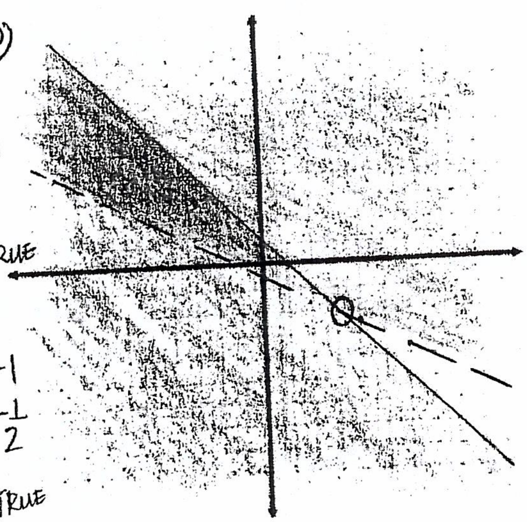
$x + 2y > -1$

$2y > -x - 1$

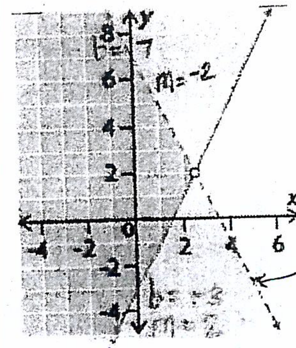
$y > \frac{-1}{2}x - \frac{1}{2}$

$0 + 0 > -1$

$0 > -1$ TRUE



47.



$y < -2x + 7$

check (0,0)

$0 < -2(0) + 7$

$0 < 7$

dashed

Solid

$y \geq 2x - 3$

check (0,0)

$0 \quad 2(0) - 3$

$0 \geq -3$

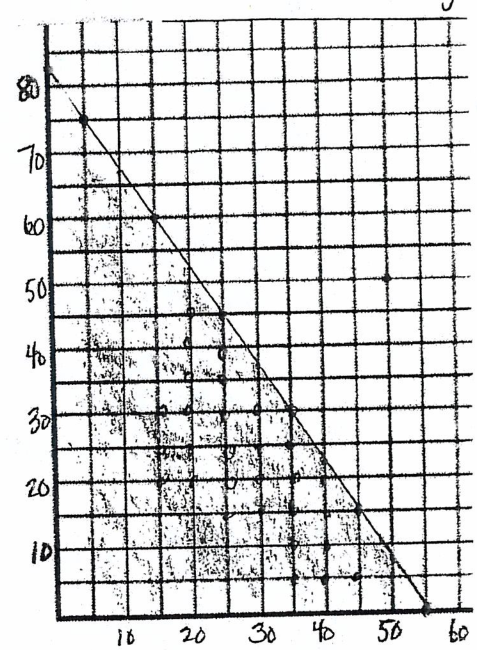
48. x = rectangular table (12 people)

a) y = circular table (8 people)

$12x + 8y \leq 660$

$\rightarrow y \leq -\frac{3}{2}x + 82.5$

restricted to whole #'s. So stipple the boundary line & sol'n region



b) as close to the same # of tables as possible (30, 30) would work but it wouldn't maximize seating. (33, 33) would maximize.

$33(12) + 33(8) =$

$396 + 264 = 660.$

- Nick's Soup -

49. x = peppers $\quad x, y \in \mathbb{R}$

a) y = tomatoes

$x \geq 2y$

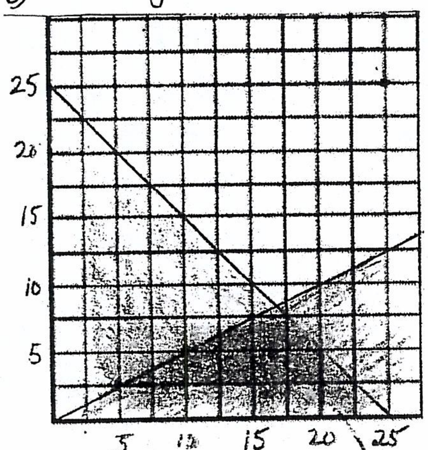
$x + y \leq 25$

$y \leq -x + 25$

$2y \leq x$

$y \leq \frac{1}{2}x$

b)



combinations (peppers, tomatoes) in the sol'n region

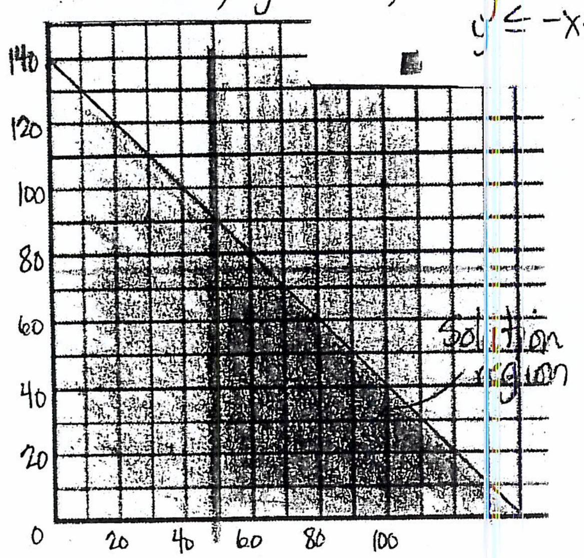
(10, 5) (15, 5) (20, 5)

50. a) x = ribbon flowers $\quad x, y \in W$

y = crepe paper rosettes

$x \geq 50$, $y \leq 75$, $x + y \leq 140$

$y \leq -x + 140$



50. Continued..... Minimizing Time.

Objective Function : $T = 6x + 9y$

Vertices : (50,0) (50,75) (65,75) (140,0)

$T = 6(50) + 9(0)$ $\quad T = 6(50) + 9(75)$ $\quad T = 6(65) + 9(75)$

$T = 300$ min $\quad T = 975$ min $\quad T = 1065$ min

minimum Time would be 50 ribbon flowers and no crepe paper rosettes.

It would take 300 minutes or 5 hours.

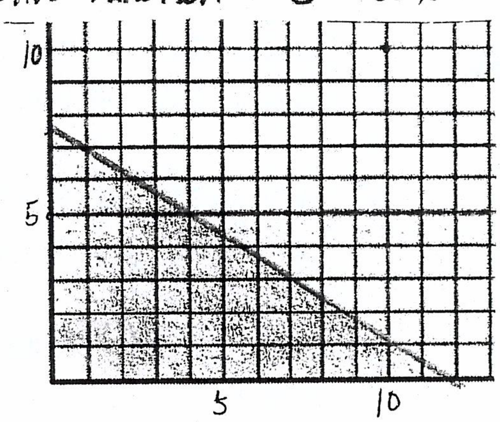
51. x = vans $\quad y$ = mini buses $\quad x, y \in W$

$y \leq 5$, $\quad 10x + 16y \leq 120$

$16y \leq -10x + 120$

$y \leq -\frac{5}{8}x + 7.5$

objective Function : $C = 550x + 730y$



Vertices. Vans, minibuses

(0,5)	(4,5)	(12,0)
$550(0) + 730(5)$	$550(4) + 730(5)$	$550(12) + 730(0)$
$C = 3650$	$C = 5850$	$C = 6600$

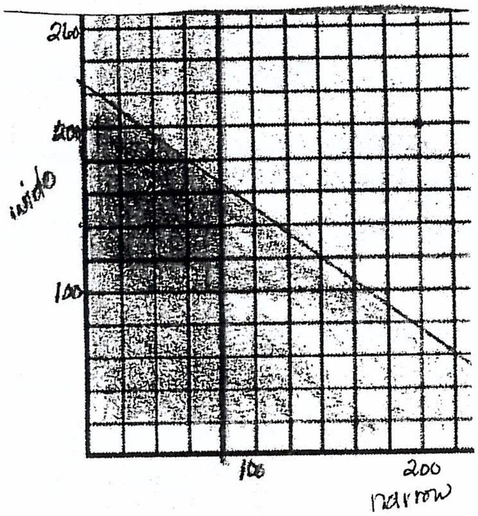
maximum value comes with 12 minivans and no minibuses \$6600. transporting 120 people.

52. n = narrow boards $n, w \in W$ convert 50 y x 36
w = wide boards

$w \geq 100$, $n \leq 80$, $6n + 8w \leq 1800$

$8w \leq -6n + 1800$

$w \leq -\frac{3}{4}n + 225$



OBJ FUN: $3.56n + 4.36w = C$

Vertices: (narrow, wide)

(0,100) \$436.00   (0,225) \$981.00

(80,100) \$720.00   (80,165) \$1004.20

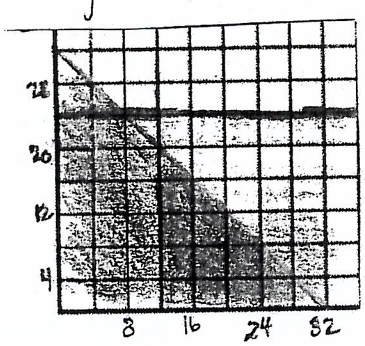
see pg 339 for more

Minimum Cost: 0 narrow/100 wide = \$436.00
Maximum Cost: 80 narrow/165 wide = \$1004.20

53. x = job #1 pays 8.75/hr $x, y \in W$
y = job #2 pays 9.00/hr
E = earnings

$x + y \leq 32$
$x \geq 12$
$y \leq 24$

OBJ FUN =
$E = 8.75x + 9.00y$



Vertices

(12,0) \$105.00   (12,20) 105 + 180 \$285.00

(32,0) \$280.00

Maximum Earnings
12 hrs @ 8.75/hr
20 hrs @ 9.00/hr.

54. a) $x^2 - 8x = f(x)$
$x(x-8) = f(x)$

$x = 0$ $x - 8 = 0$
$x = 8$

- x intercepts are 0 and 8

$\frac{0+8}{2} = \frac{8}{2} \to 4$

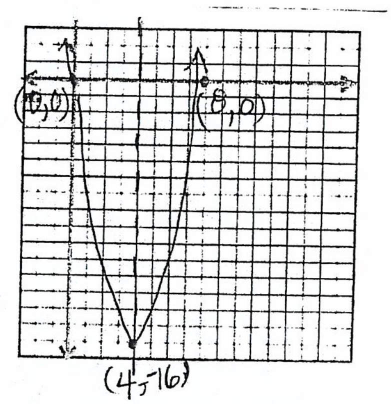
- axis of symmetry $x = 4$
vertex lies on the axis of symmetry

$4^2 - 8(4) =$
$16 - 32 = -16$

- vertex $(4, -16)$

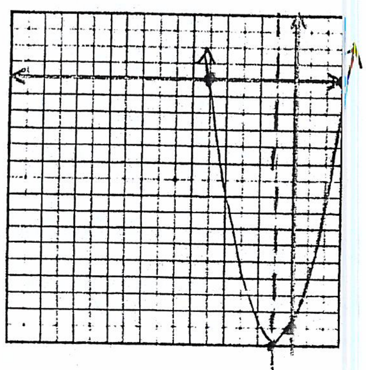
y-intercept $x = 0$
$0^2 - 8(0) = 0$

- y intercept $\to (0,0)$



Domain $x \in \mathbb{R}$
Range $y \geq -16$

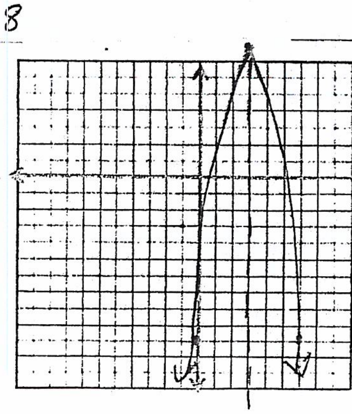
54. b) $y = x^2 + 2x - 15$
$y = (x+5)(x-3)$
$x+5=0 \quad x-3=0$
- $x=-5 \quad x=+3$
- axis of symmetry $x=-1$
$\frac{(-5+3)}{2} = \frac{-2}{2} = -1$
- $y=(-1)^2+2(-1)-15$
$y = 1-2-15$
$y=-16$
- vertex: $(-1,-16)$
- y intercept $=-15$



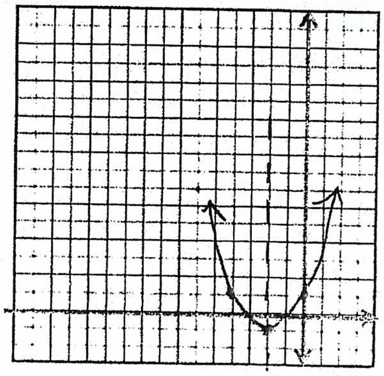
Domain $x \in \mathbb{R}$
Range $y \geq -16$

55. $y=-2(x-3)^2+8$
a) vertex $=(3,8)$
axis of symm $x=3$
y intercept $=-2(0-3)^2+8$
$-2(-3)^2+8$
$-2(9)+8$
$=-10$

$x \in \mathbb{R}$
$y \leq 8$



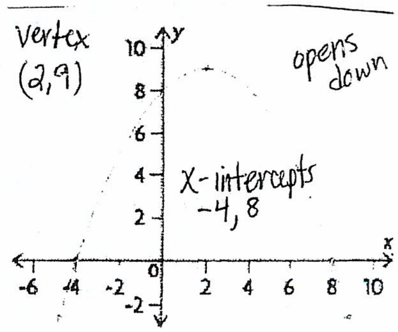
55. b) $g(x) = .5(x+2)^2-1$
vertex $=(-2,-1)$
axis of symmetry $x=-2$
y int $= .5(0+2)^2-1$
$.5(4)-1$
$2-1$
$(0,1)$



$D = x \in \mathbb{R}$
$R = y \geq -1$

56. $f(x) = 4x^2+24x+31$
axis of symmetry $x=23$
a) opens up because a is positive
b) the vertex lies on the axis of symm.
so plug it in... $4(-3)^2+24(-3)+31$
$4(9)+(-72)+31$
$36+(-72)+31 = -5$
vertex $=(-3,-5)$
c) a minimum value because it opens up.

57. vertex $(2,9)$



$y=a(x-r)(x-s)$
$9=a(2-(-4))(2-8)$
$9=a(6)(-6) \rightsquigarrow \frac{9}{-36}=\frac{-36a}{-36}$
$y=\frac{-1}{4}(x+4)(x-8)$ OR $y=-\frac{1}{4}x^2+x+8$

58. Determine the Roots (aka: the x-intercepts)

a) -3 and 3 } just by looking at graph.

b) $-\frac{1}{2}$ and 3

59. a) $(x-5)(2x+1)=0$

$x-5=0 \quad 2x+1=0$

$x=5 \quad x=-\frac{1}{2}$

b) $x^2-4x-32=0$

$(x-8)(x+4)=0$

$x=8 \quad x=-4$

c) $3x^2-10x-8=0$

$(3x+2)(x-4)=0$

$x=-\frac{2}{3} \quad x=4$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

d) $x^2-6-10=0$! can't factor so use Quad Formula

$a=1 \quad b=-6 \quad c=-10$

$\frac{-(-6) \pm \sqrt{36-4(1)(-10)}}{2} \Rightarrow \frac{6 \pm \sqrt{76}}{2}$

$= \frac{6 \pm 2\sqrt{19}}{2} \Rightarrow 3 \pm \sqrt{19}$

e) $2(x-3)^2-8=0$

$2(x^2-6x+9)-8=0$

$2x^2-12x+18-8=0$

$2x^2-12x+10=0$

$2(x^2-6x+5)=0$

$(x-5)(x-1)=0$

$x=5 \quad x=1$

f) $\overset{(a)}{1.5x^2} - \overset{(b)}{6.1x} + \overset{(c)}{1.1} = 0$

$\frac{-(-6.1) \pm \sqrt{(-6.1)^2-4(1.5)(1.1)}}{2(1.5)}$

$\frac{6.1 \pm \sqrt{37.21-6.6}}{3}$

$\frac{6.1 \pm \sqrt{30.61}}{3}$

$\frac{6.1+5.53}{3} = 3.88$

$\frac{6.1-5.53}{3} = .19$

60. $h(t) = 4500 - 5t^2$

a) $h = 4500 - 5(5)^2$

$h = 4500 - 125$

$h = 4375$ m

b) $1500 = 4500 - 5t$

$5t^2 - 3000 = 0$

$5(t^2 - 600) = 0$

$\sqrt{t^2} = \sqrt{600}$

$t = 24.5$ s

61. y-int $= (0,-4)$ Vertex $(3,-7)$

$y = a(x-h)^2 + k$

$-4 = a(0-(-3))^2 + (-7)$

$-4 = a(9) - 7$

$\frac{3}{9} = \frac{9a}{9}$

$a = \frac{1}{3}$

$y = \frac{1}{3}(x+3)^2 - 7$

$y = \frac{1}{3}(x^2+6x+9) - 7$

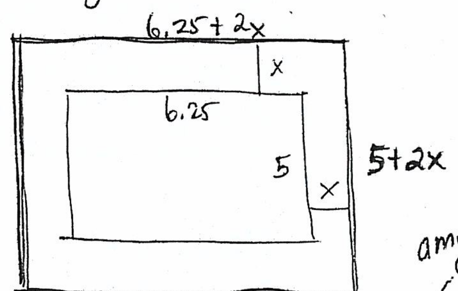
$y = \frac{1}{3}x^2 + 2x + 3 - 7$

$y = \frac{1}{3}x^2 + 2x - 4$

62.

Area of path = Big▭ − Small▭

$A = l \times w$



amount of crushed rock

$(6.25+2x)(5+2x) - (6.25)(5) = 6 \text{ m}^2$

$\cancel{31.25} + 10x + 12.5x + 4x^2 - 31.25 = 6$

$4x^2 + 22.5x - 6 = 0$

$a = 4$

$b = 22.5$

$c = -6$

$\frac{-22.5 \pm \sqrt{(22.5)^2 - 4(4)(-6)}}{2(4)}$

$\frac{-22.5 \pm \sqrt{602.25}}{8}$

$\frac{-22.5 \pm 24.5}{8}$

$\frac{2}{8} = .25$

$\frac{-47}{8} = -5.9$ impossible

The border should be .25 m wide.