2.4 Angle Properties in Polygons (Concepts #19 and 20)

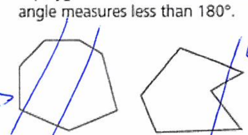
Types of Polygons:

convex polygon

A polygon in which each interior angle measures less than 180°.

line test

only crosses twice



convex

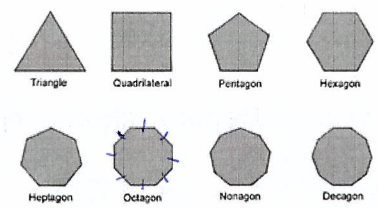
non-convex (concave)

← Crosses 3 times.

Regular Polygon

- a polygon whose... sides are all equal
- a polygon whose... interior angles are all equal.

Regular polygons



What is the interior angle sum of **any triangle**? __180°__ (we proved this last section)

What will the 4 interior angles of any quadrilateral always add to? The 5 interior angles of a pentagon? Let's investigate:

* Draw Shapes *

Polygon	# of Sides	# of Triangles	Sum of Interior Angle Measures
Triangle	3	1	180
Quadrilateral	4	2	$2(180°) = 360°$
Pentagon	5	3	$3(180°) = 540°$
Hexagon	6	4	$4(180°) = 720°$
Heptagon	7	5	$5(180°) = 900°$
Octagon	8	6	$6(180°) = 1080°$

Talk about why?

*because a quadrilateral is made up of two triangles, and one triangle has a sum of interior angle measures so, $2 \times 180° = 360°$ will be the sum of interior angles.

How can we find the **sum** of the interior angles based on the **number of sides** of a polygon has?

Let n = # of sides

Sum of interior ∠'s = $(n-2)(180°)$

Example 1: a) What would be the sum of the measures of the interior angles of a regular dodecagon (12-sides)?

$$\text{Sum of interior } \angle\text{'s} = (12-2)(180^\circ)$$
$$= (10)(180^\circ)$$
$$= 1800^\circ$$

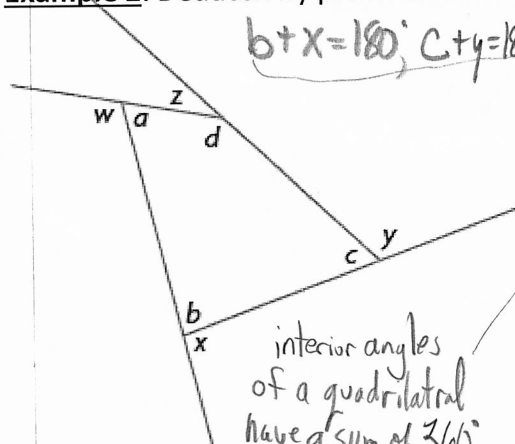
b) Determine the measure of each interior angle of a regular dodecagon?

$$\text{One interior angle} = \frac{1800}{12} = 150^\circ$$

In a regular polygon all interior angles are equal

Example 2: Deductively prove that the sum of the exterior angles of any polygon will be 360^0

$b+x=180^\circ;\ c+y=180^\circ;\ z+d=180,\ w+a=180^\circ$ ⇒ Supplementary angles (Straight lines)



interior angles of a quadrilateral have a sum of 360°

$$(b+x)+(c+y)+(z+d)+(w+a) = 4(180^\circ)$$
$$(a+b+c+d)+(x+y+w+z) = 720^\circ$$
$$360^\circ + x+y+w+z = 720^\circ \Rightarrow \text{substitute}$$
$$x+y+w+z = 360^\circ \Rightarrow \text{Subtracted } 360^\circ \text{ from both sides.}$$

In Summary

Key Idea

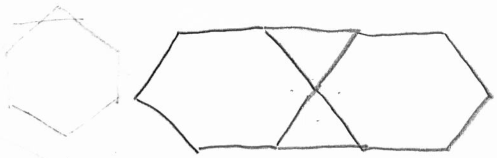
- You can prove properties of angles in polygons using other angle properties that have already been proved.

Need to Know

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180^\circ(n-2)$.
- The measure of each interior angle of a regular polygon is $\frac{180^\circ(n-2)}{n}$.
- The sum of the measures of the exterior angles of any convex polygon is 360°.

Remind: * Regular polygons have all equal sides and interior angles *

(Concept #20) Example 3: Bob is tiling his floor. He uses regular hexagons and regular triangles. The side length of a triangle is equal to the side length of a hexagonal tile. Can he tile the floor without leaving any gaps between tiles.?



* Consider the vertices where the shapes meet. Each vertex is formed by 2 interior angles of a regular hexagon and 2 interior angles of a triangle.

* To fit in a perfect pattern, the 4 angles must add to 360°.

Step 1: Find the measure of each interior angle of a regular hexagon and triangle.

$n = 6$

Hexagon $= \frac{18(6-2)}{8}$ Triangle $= 60°$

$= 120°$

Step 2: Add all angles at any vertex

$60 + 60 + 120 + 120 = 360°$

∴ He can tile the floor without any gaps

Example 4: Kieran drew a 14 sided convex polygon. One of the interior angle measures 155°, Is it a regular polygon? No

interior ∡'s $= \frac{180(n-2)}{n}$

$n \times 155° = \frac{180(n-2)}{n} \times n$

$155n = 180n - 360$

$-180n \qquad -180n$

$\frac{-25n}{-25} = \frac{-360}{-25}$

$n = 14.4 \Rightarrow$ No, it is not a regular polygon it is a 14.4 sided polygon which the # of sides needs to be a whole #.

Assignment Pg 99 #1,2,3,7,10,13,16,18