# 1.4 Proving Conjectures : Deductive Reasoning (Concept # 15)

Let's review what we already know:

**Inductive Reasoning:** is a type of reasoning in which we arrive at a conclusion, generalization or educated guess based on experience, observation or patterns

The conclusion, generalization or education guess which is arrived at by using inductive reasoning is called a **CONJECTURE.** Conjectures may or may not be true.

**Example #1:** Consider the following

$2^2 + 1 = 5$ (prime number)

$4^2 + 1 = 17$ (prime number)

$6^2 + 1 = 37$ (prime number)

$10^2 + 1 = 101$ (prime number)

$20^2 + 1 = 401$ (prime number)

$36^2 + 1 = 1297$ (prime number)

a) Write a conjecture based on the information to the left:

- one more than a squared even number equals a prime number
- The sum of an even number squared an 1 is a prim #

b) What type of reasoning is the conjecture based on?

Inductive, I cameup with a conclusion based on a pattern with a few examples

c) Provide a counter example to show the conjecture in example 1 is false.

$8^2 + 1 = 65$ (composite #)

Note: "A counterexample is an example which shows that a conjecture is false"

**Deductive Reasoning:** A specific conclusion through logical reasoning by starting with general assumptions that are known to be valid. A type of reasoning based on things you already know to be true

**Proof:** A mathematical argument showing that a statement is valid in all cases, or that no counterexamples exist.

We will be doing some forms of mathematical proofs in this unit. A few things that might be helpful to know about some of the algebra we will be encountering:

Let n = a particular number

If I wanted to talk about a version of the number "n" that was an even number , what could I call it algebraically? Why?

Even # = $2n$, because all even #'s are divisible by 2 or all even numbers have a factor of 2.

If I wanted to talk about a version of the number "n" that was odd, what could I call it algebraically? Why?

Odd # = $2n+1$, an odd number is always one larger than an even #.

If I wanted to talk about the next consecutive number after "n", what could I call it algebraically

$\ldots n-2, n-1, n, n+1, n+2, n+3 \ldots$

one # smaller than n

one # bigger than n

**Example #2:** Jon discovered a pattern when adding integers

$1+2+3+4+5 = 15$

$(-15) + (-14) + (-13) + (-12) + (-11) = -65$

$(-3) + (-2) + (-1) + (0) + 1 = -5$

Jon's Claim: Whenever you add five consecutive integers, the sum is always 5 times the median of the numbers

"What kid of reasoning did Jon use?" Inductive

"What is a median?" The median is the number that the middle number in a set of numbers. If there are a even set of numbers, take the average of the middle two to find median.

**Prove his conjecture:**

$\boxed{\text{Step 1}}$ Show that all 3 examples are true

$1+2+\underset{\text{median}}{3}+4+5=15 \qquad 5\times 3=15$ True ✓

$(-15)+(-14)(-13)+(-12)+(-11)=-65 \qquad (-13)\times 5=-65$ True ✓

median

$(-3)+(-2)+(-1)+0+1=-5 \qquad (-1)\times 5=-5$ true ✓

median

$\boxed{\text{Step 2}}$ Prove all cases algebraically

Let $x$ = any integer

Let $S$ = The sum of 5 consecutive integers

Note: "We will let $x$ be the integer that happens to be the median"

$$S=(x-2)+(x-1)+\underset{\text{Median \#}}{x}+(x+1)+(x+2)$$

$$S=5x-3+3$$

"Combine Like terms"

$$S=5x$$

Since S (the sum) = 5 times the integer that is the median we have algebraically proven the conjecture to be true

Note: Even #'s are 2, 4, 6, 8 ... (#'s that are divisible by 2)
Odd #'s are 1, 3, 5, 7 ... (not divisible by 2)
Product means to multiply

**Example #3:** Prove that the product of an even integer and an odd integer is always even.

"Create a few examples to see the pattern"

$\rightarrow (2)(3) = 6$
$\rightarrow (-10)(5) = -50$
$\rightarrow (-8)(-8) = +64$

this is not a proof, this is merely inductive reasoning to show specific cases

"Prove algebraically the conjecture"

Let $2m$ = an even integer
Let $2n+1$ = an odd integer

$2m(2n+1)$ → Product of an even # and odd #
$= 4mn + 2m$ → A trick to proving a # is even is to show is can be written as 2(something)
$= 2(2mn + m)$

Our algebraic expression will always be even because we have proven 2 is a factor.

**Example #4:** Prove that the difference between consecutive perfect squares is always an odd number.

Let $x$ = any integer
Let $D$ = the difference between consecutive perfect squares

$$D = \underbrace{(x+1)^2}_{\text{the next squared \#}} - \underbrace{x^2}_{\text{a square \#}}$$

$D = (x+1)(x+1) - x^2$
$D = x^2 + 1x + 1x + 1 - x^2$
$D = 2x + 1$ → because the value of D is in the form of an odd integer, we have proven our conjecture to be true.

**Example #5:** Prove that any three digit number is divisible by five when the last digit in the number is divisible by five.

Let abc = represent a 3 digit number

hundreds tens ones

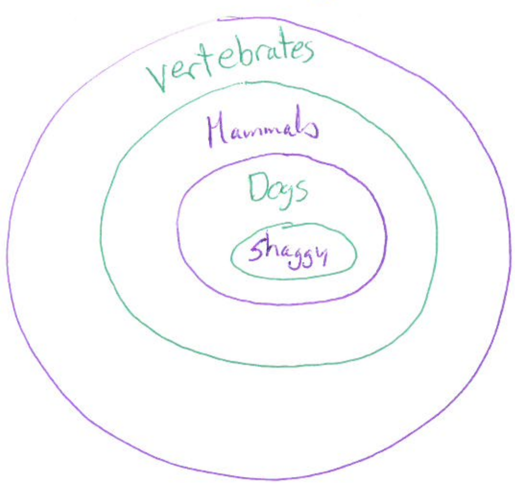
$$abc = 100a + 10b + 1c$$ ← the last digit is divisible by 5.

$$abc = 5(20a + 2b) + c$$

We have proven that 5 is a factor of the number "abc" therefore any 3 digit number is divisible by 5 when the last digit is divisible by 5.

**Example #6** Use a VENN Diagram to prove the following: All dogs are mammals. All mammals are vertebrates. Shaggy is a dog. What can be deduced about Shaggy?

We can use a Venn diagram to prove




Assignment 1.4 ( Concept #15) Pg 31 #1-5, 7, 8, 10 , 11 , 13, 20