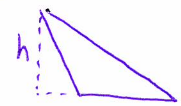
1.2/3 Using reasoning to find a counterexample to a conjecture (Concept #15 and 16)

Example 1: Show that each statement is false by finding a counter example

a) A number that is not negative is positive 0 is a number that is not negative, and is not positive

b) The height of a triangle lies inside the triangle The height of the triangle can lie outside the triangle

c) The square root of a number is always less than the number



$\sqrt{0.01} = 0.1$

$0.1 > 0.01$

Example2: Harry claims that if opposite sides of a quadrilateral are the same length, the quadrilateral is a rectangle. Do you agree or disagree? Justify your decision.

Disagree, the quadrilateral could be a square. A square has 4 equal sides therefore opposite sides are equal. Also a rhombus and parralellogram.

Example3: Amy made the following conjecture. When any number is multiplied by itself, the product will be greater than this starting number. Do you agree or disagree with Amy's conjecture? If you disagree provide a counter example and revise her conjecture to be improved.

Disagree, Counterexample

$1 \times 1 = 1$

1 is not greater than 1 it equals 1.

Question: If you can't find a counter example can you be certain that one does not exist?

Even if you can't find a counter example, you cannot be certain that there is not one. Any supporting evidence you develop while searching for a counterexample, however, does increase the likelihood that the conjecture is true.

Pg 22 #1, 2, 3-9, 16 and 17

Review 1.4 Deductive Reasoning (Concept #15)

a) Prove that the sum of a two digit number and the number formed by reversing its digits will always be divisible by 11.

Let ab = two digit number

Let S = Sum

The original # can be written as $10a + b$

The # formed by reversing the digits can be written as $10b + a$

$$S = (10a+b) + (10b+a)$$
$$S = \cancel{2}11a + 11b$$
$$S = 11(a+b)$$

Therefore 11 is a factor of the sum of a two digit # and the # formed by reversing the digits which means it is always divisible by 11.

b) Prove that the sum of the squares of two consecutive odd integers is even.

Let $2x+1$ = odd integer

Let x = any integer

Let $2x+3$ = next consecutive odd integer

Let S = Sum

$$S = (2x+1)^2 + (2x+3)^2$$
$$= (2x+1)(2x+1) + (2x+3)(2x+3)$$
$$= 4x^2 + 4x + 1 + 4x^2 + 12x + 9$$
$$= 8x^2 + 16x + 10$$
$$= 2(4x^2 + 8x + 5)$$

Since 2 is a factor of the Sum it will always be even.

c) Prove that the sum of any three consecutive multiples of 3 is divisible by 9

Let K = any integer

Let 3K = # that is a multiple of 3.

Let S = Sum.

$$3K + 3(K+1) + 3(K+2) = S$$
$$3K + 3K + 3 + 3K + 6 = S$$
$$9K + 9 = S$$
$$9(K+1) = S$$

9 is a factor of the sum, therefore the sum is divisible by 9.